FADING DISPERSIVE CHANNEL ESTIMATION AND EQUALIZATION USING CIRCULATING FILTERS

By

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INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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CERTIFICATE

This is to certify that the thesis entitled 'FADING DISPERSIVE CHANNEL ESTIMATION AND EQUALIZATION USING CIRCULATING FILTERS' by Naresh Chandra Vyas has been carried out under my supervision and has not been submitted elsewhere for a degree.

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Kanpur,
August, 1978

– Naresh Chandra Vyas

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ABSTRACT

An adaptive decision-directed scheme using circulating filters is described for the estimation of the impulse response of fading dispersive channels. Theoretical and computer simulation results show that more than 12 dB of SNR improvement can be realized.

Incorporating the above scheme and taking advantage of the skewed nature of the delay power spectrum of troposcatter channels, a decision feedback equalization scheme which can implement matched filtering with a small loss of the received signal energy is considered and its performance evaluated by computer simulation. Also studied is the performance of the circulating filters in Viterbi algorithm-ML sequential decoder. Simulation results indicate that a low probability of error performance can be obtained.

CHAPTER 1

INTRODUCTION

This thesis considers the problem of adaptive estimation and equalization of the fading dispersive channels in the context of high speed binary data transmission. The objective of the study is to develop an efficient channel estimation technique and to observe how it performs in some representative examples of channel equalization schemes.

1.1 Background

Most of the communication channels are band limited and data transmission over them results in symbol dispersion and hence in intersymbol interference. Also corrupting the received signals are the thermal and other noises. As a result, the received signals bear almost no resemblance to the transmitted signals. Efforts are made to minimize these effects at the transmitter end by optimizing the pulse shape [Cha-50, TS-64] so that it suffers least energy loss in passing through the channel (signal shaping) and at the receiver end by compensating for the channel dispersion (channel equalization) [LSW - 68]. A number of theoretical schemes have been proposed [Vit-67, AF - 70] to permit high performance data communication over these channels. A major problem faced in practical application of these schemes is

that channel characteristics are rarely known in sufficiently accurate detail to allow a design that will result in near optimum performance. The direct approach of making channel measurements and then designing the appropriate receiver is impractical in most cases. The technique then adopted is to use adaptive receivers that are capable of adjusting automatically to accommodate changing channel conditions.

The problem is more acute in tropo-applications where the channel is fading dispersive [Bel-63, SBS-66]. The channel characteristics are in fact the major element to be evaluated in the study of any communication scheme for troposcatter channels. The performance of any data communication system for tropo applications depends to a large extent on how well it is able to adapt to channel variations.

The need for accurate channel characterization in a high performance receiver scheme necessiates inclusion of a well designed channel estimator in the receiver structure. The test-signal and decision-directed methods [LSW-68] are the two techniques normally employed for channel estimation. In the first case a separate test signal, say a known data pattern, sent over the channel is used in extracting information about the channel response. The test signal techniques are inaccurate and costly as compared to the decision directed techniques. In the decision directed method receiver-decisions as to the past transmitted symbols are

used in extracting channel information from the received signal and separate test signals are not needed. The scheme works well so long as the decision errors occur only once in a while. In case of frequent decision errors the channel estimates will be inaccurate and equalizer adaption will be affected. This may lead to more decision errors. In extreme situations, long bursts of errors may result in complete loss of channel acquisition, but in practical receivers where probability of error is of the order of 10^{-5} or 10^{-6} this event is really rare.

The equalization techniques normally employed are linear processing and nonlinear or recursive processing of the received signal. In linear processing a matched filter followed by a transversal filter is used [LSW-68].The adjustable parameters are the tap gain factors. A number of strategies like zero forcing method [Luc-65, LSW-68] and steepest descent algorithm [LSW-68] have been proposed to adaptively adjust these tap weights. The performance of these linear techniques is significantly inferior to the recursive equalization techniques like decision feedback equalization [Aus-67, GBS-71] and Viterbi decoding [Vit-67, Vit-71]. Of these, the Viterbi decoding has been shown to be optimum structure [For-72, Kob-71].

The decision feedback equalization is similar to the transversal equalizer in that both have a matched filter followed by a tapped delay line. However it makes use of the previous decisions to coherently subtract the intersymbol interference caused by the previously detected symbols. This is done by passing the past decisions through the feedback TDL. The MF and forward TDL are used to minimize the effects of additive noise and future symbol interference.

The Viterbi algorithm is a sequential decoding technique. It calculates the likelihood functions of various allowable sequences recursively and selects the one with maximum likelihood as the decoded sequence.

1.2 Previous Related Work

The 1928 paper by Nyquist [Nyq-28] is a classic paper in the data transmission field due to its lasting influence on the data system theory and design. The problem he considered was to find conditions on the transmitted pulse shape so that the samples of the received signal are identical to the message sequence in the absence of ISI and noise. The pulse shaping problem is further studied in detail in the subsequent papers by Gabon [Gab-46], Chalk [Cha-50], Tufts and Schnidman [TS-64], DiToro and Steiglitz [DS-65], Gerst and Diamond [GD-61] and others and also in books like Bennet and Davey [BD-65] and Lucky, Salz and Weldon [LSW-68].

The joint optimization of transmitter and receiver to minimize the effects of ISI and noise has been considered by Tufts [Tuf-65], Smith [Smi-65], Berger and Tufts [BT-67], Hanster [Han-71], Ericson [Eri-73] and others.

The work on linear nonrecursive equalizers using tapped delay line dates back to Kalmann [Kal-40] who proposed use of transversal filters for signal processing. Ericson [Eri-71] has shown that for any reasonable criteria a matched filter-TDL combination is the expected receiver structure. Adaptive versions of linear equalization techniques have been given by Lucky [Luc-65, Luc-66], Gersho [Ger-69] and Proakis and Miller [PM-69].

The nonlinear techniques have been studied in two different categories. The decision feedback equalization has been considered in detail by Austin [Aus-67], Brady [Bra-70], George, Bowen and Storey [GBS-71] and Monsen [Mon-71, Mon-73, Mon-74 and MR-73]. Brady and Monsen have studied the technique for tropo applications. In the second categories optimum techniques have been considered, the important ones being statistical detection technique described by Abend and Fritchman [AF-70] and Viterbi decoding first described by Viterbi [Vit-67]. The Viterbi decoding has been subsequently studied in detail by Viterbi [Vit-71], Forney [For-72, For-74] and others. A bibliography of work

in Viterbi decoding field upto 1973 is contained in [For-73].

1.3 Organization of the Thesis

In this thesis an efficient channel estimation scheme is developed and its performance for two nonlinear equalization schemes; the decision feedback equalization and Viterbi decoding, for fading dispersive channels is considered. In Chapter 2 the troposcatter channels are modelled and an estimation scheme using circulating filters which is suitable for tropo-applications is considered. An expression for noise suppression due to circulating filters is derived and simulation results indicating the estimation performance are given. In Chapter 3 a DFE structure for troposcatter channels which requires explicit knowledge about the channel response is obtained and the performance of the system using circulating filters for channel estimation is determined by computer simulation. In Chapter 4 optimum ML estimator structure using Viterbi algorithm is considered. A receiver structure using circulating filter estimators is derived and its performance is studied by way of computer simulation. Chapter 5 concludes the thesis by discussing the results obtained and exploring the possibilities of further work in the field. In Appendix 1 the details of the simulations are given and in Appendix 2 the computer programs for these simulations are given.

CHAPTER 2

TROPOSCATTER CHANNEL ESTIMATION USING CIRCULATING FILTERS

Signals received through practical communication channels bear little resemblance to the transmitted signals. Efficient recovery of transmitted information from such received signals depends not only on the choice of signalling waveforms transmitted and signal processing techniques employed in the receiver but also on the knowledge of the channel characteristics. Thus, modelling and estimation of channel characteristics constitute important aspects in the design of communication systems.

In this chapter we consider a scheme of adaptively estimating the low pass equivalent complex impulse response of a fading time dispersive channel, such as the troposcatter channel, over which a binary phase shift keyed (BPSK) signal is transmitted. In Section 1 we introduce a fairly general model for a fading time dispersive channel . In Section 2 we formulate the channel estimation problem and indicate that the opt. solution implied by such an approach is not practical. Assuming the availability of error free decisions on past symbols we then illustrate in Section 3 the possibility of using a circulating filter for estimating the channel impulse response. In Section 4, we relax the assumption on

intersymbol interference and discuss a scheme of estimation with circulating filters to obtain essentially noise free estimates of the channel impulse response. We also obtain an expression for noise reduction attainable in the estimator. In Section 5 we present some simulation results relating to convergence of the estimates in the presence of symbol decision errors and time-jitter.

2.1 A Model for Fading Dispersive Channels

Following Kailath [Kai-60] and Bello [Bel-63, Bel-69] we assume that a fading dispersive communication channel can be modelled as a random linear time-varying filter with low pass equivalent, possibly complex, impulse response $h(t,\tau)$ which is the response at time t due to an impulse applied τ sec. earlier. A troposcatter channel which is of major interest to this thesis provides a practical example of a fading dispersive channel. Empirical evidence as well as central limit theorem based arguments [Bel-69, SBS-66] suggest that the impulse response $h(t,\tau)$ of a tropo channel may adequately be considered as a zero mean complex Gaussian process. Further under the usually realistic assumption of wide sense stationarity and uncorrelated scattering such a channel is completely characterized by its path-gain correlation function, [Bel-69, SBS-66],

$$E[h(t,\tau)h^{*}(t+\varepsilon,\eta)] = Q(\varepsilon,\tau)\ \delta(\eta-\tau)$$

Moreover, when the fading rate is small compared to the information transmission rate the channel may be considered to be quasi time-invariant. The delay power spectrum $Q(0,\tau)$, often denoted by $Q(\tau)$, of the channel is then

$$E[h(\tau)h^{*}(\eta)] = Q(\tau)\ \delta(\eta-\tau)$$

and represents power spectrum of the channel transfer function in frequency domain.

The delay power spectrum of a typical tropo-link, shown in Fig. 2.1, can be reasonably represented by the empirical expression

$$Q(\tau) = A\tau\ e^{-\alpha\tau} \qquad \tau \geq 0$$

2.1.1 BPSK Transmission over Tropo-channel

The baseband model of data system is shown in Fig. 2.2.

The low -pass equivalent impulse response $h(t)$ of the tropo-channel is finite, $h(t) = 0$ for $t \geq nT$ and has n chips

$$h(t) = h_0(t) + h_1(t-T) + h_2(t-2T) + \ldots\ h_{n-1}(t-\overline{n-1}T)$$

where the chip $h_i(t)$; $i = 0,1, \ldots. (n-1)$ is nonzero over the interval $(0,T]$.

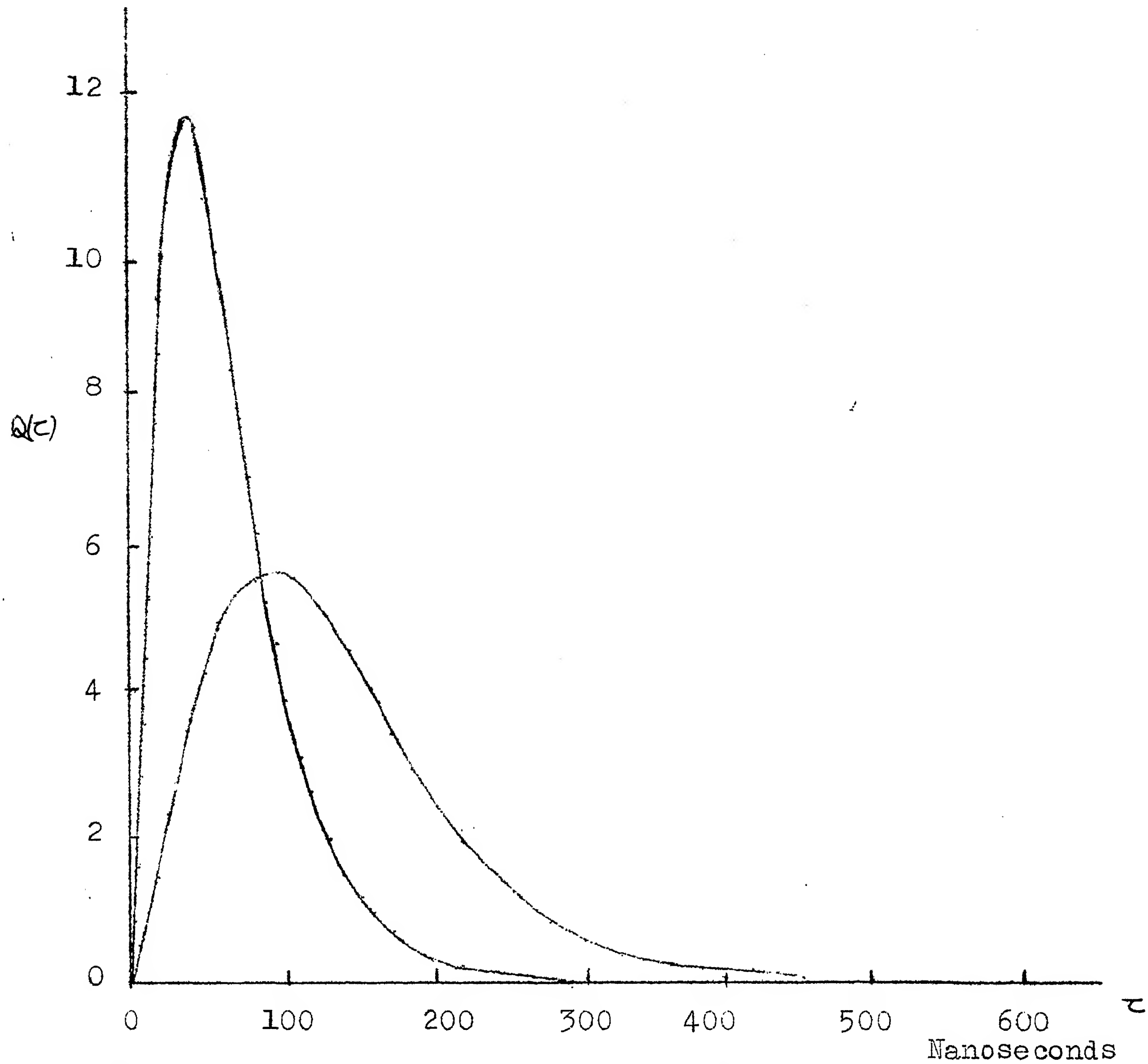


Fig. 2.1 Delay power spectra for the short and long channels

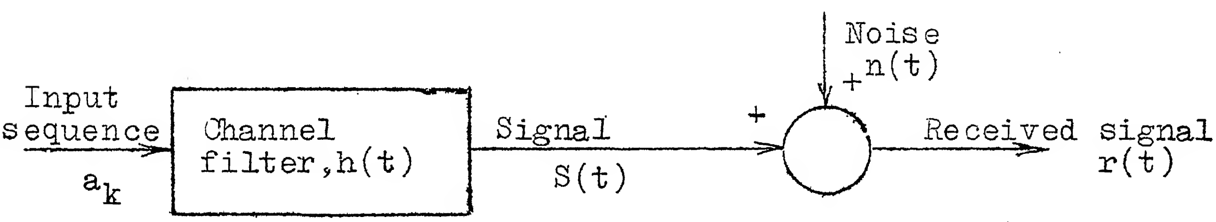


Fig. 2.2 Baseband model for a communications channel

The channel pulse response or composite channel impulse response $h^c(t)$ is defined as

$$h^c(t) \doteq h(t) * p(t)$$

where p(t) is unit pulse of duration T.

The input to the channel is a sequence a_k of randomly selected equally likely pulses of amplitude $\pm$ 1. The channel output is

$$r(t) = \sum_k a_k \, h(t-kT) + n(t)$$

or,
$$r(t) = \sum_k b_k h^c(t-kT) + n(t)$$

where b_k is input sequence of $\pm$ 1 impulses and n(t) is zeromean complex white Gaussian noise of variance σ^2.

2.2 Channel Estimation Problem

The channel estimation problem is given the received or observed signal

$$r(t) = \sum_k a_k \, h(t-kT) + n(t)$$

determine the channel impulse response h(t).

Optimum solution to the channel estimation problem can be obtained by techniques like MAP estimation and Kalman

filtering. These methods are considered in detail in VanTrees, Part III. Since both the input sequence a_k and channel response h(t) are unknown, the state equation formulation of the estimation problem suitable for Kalman filtering is nonlinear. The Kalman filter is unable to treat nonlinear equations per se and quasi-linearization assumption is to be made. The estimator structure is quite complex and the technique is not suitable for adaptive channel estimation.

However, if error free decisions about symbols a_k are available simpler channel estimation schemes can be derived. This we do using circulating filters in the next section.

2.3 Channel Estimation Using Circulating Filters

In this section we consider estimation of a non-dispersive channel. The received signal in the interval k over such a channel is

$$r_k(t) = a_k\, h(t) + n_k(t)$$

In the absence of noise

$$r_k(t) = a_k\, h(t)$$

and h(t) can be determined by correlating $r_k(t)$ with a_k

$$h(t) = a_k\, r_k(t)$$

as

$$a_k^2 = 1.$$

In the general case a noisy estimate $h'(t)$ of $h(t)$ is obtained

$$h'(t) = a_k r_k(t) = a_k[a_k h(t) + n_k(t)]$$
$$= h(t) + e_k(t)$$

where $e_k(t)$ are uncorrelated zero-mean random variables with a variance ε^2.

Essentially noiseless estimates $\bar{h}(t)$ can be obtained by averaging $h'(t)$ over several bits. This is done in a circulating filter by coherently adding its input in successive intervals. The circulating filter scheme is shown in Fig. 2.3. For a feedback path gain f, the signal at the output of the circulating filter is

$$z(t) = h'(t) + fh'(t) + f^2h'(t) + \ldots + f^kh'(t)$$
$$= x(t) + y(t)$$

where, for sufficiently large k,

$$x(t) = h(t)\,[1 + f + f^2 + \ldots + f^k] = \frac{h(t)}{1-f}$$

and $\quad y(t) = e_o + fe_1 + f^2e_2 + \ldots. + f^k\, e_k.$

The variance φ^2 of $y(t)$ is given by

$$\varphi^2 = \varepsilon^2[1 + f^2 + f^4 + \ldots.\ f^{2k}] = \frac{\varepsilon^2}{1-f^2}$$

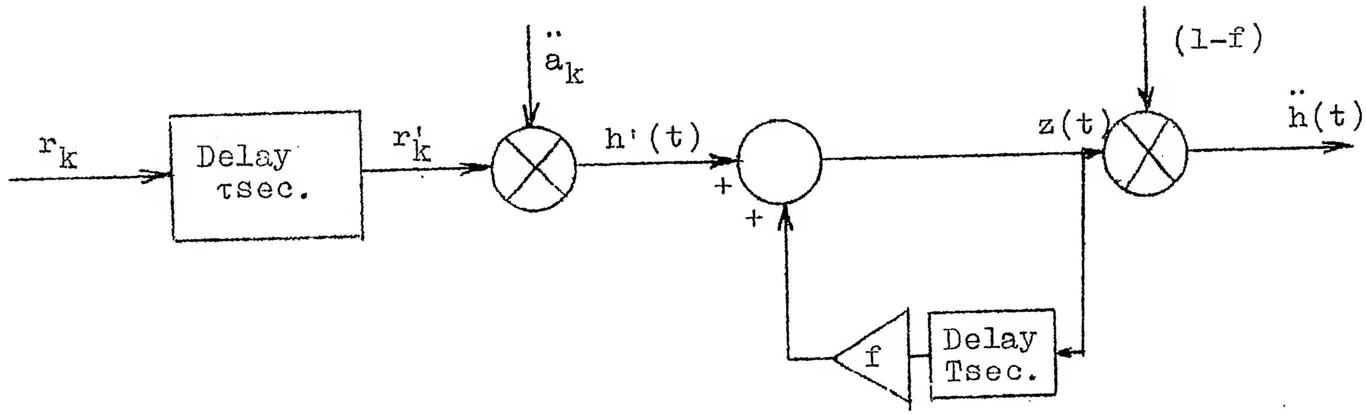


Fig. 2.3 Channel estimation using circulating filter

The channel estimate is

$$\ddot{h}(t) = (1-f)\ z(t)$$

$$= h(t) + (1-f)\ y(t)$$

The noise power is suppressed by a factor

$$\frac{(1-f)^2\ \varphi^2}{\sigma^2} = \frac{1-f}{1+f}$$

and the corrsponding SNR improvement is by a factor $\frac{1+f}{1-f}$.

For a feedback gain of 0.9, a substantial noise suppression of 12.8 dB is achieved. Thus a channel estimator using circulating filters is capable of giving essentially noiseless estimates in the absence of ISI.

In the next section we extend the concepts of channel estimation using circulating filters to channels with ISI. We consider a more realistic situation where the input symbols are not known but their almost error-free estimates are available.

2.4 Estimation of Dispersive Channels

In this section we consider estimation of a channel whose pulse response $h^c(t)$ extends to (n+1) symbols durations. The (n+1) chips of the pulse response

$$h^c(t) = \sum_{i=0}^{n} h_i(t - iT)$$

are estimated by (n+1) circulating filters one for each chip. The schematic diagram of the channel estimation scheme is shown in Fig. 2.4.

The received signal in the k^{th} interval is

$$r_k(t) = \sum_{i=0}^{n} b_{k-i}\, h_i^c(t) + n_k(t)$$

In estimating j^{th} chip $h_j^c(t)$; $j = 0,1, \ldots .\ n$ in the j^{th} filter the interference contribution due to chips $h_i^c(t)$, $i \neq j$ is removed from $r_k(t)$ using the symbol estimates $\ddot{b}_{k-i}$; $i \neq j$ and the estimates $\ddot{h}_i^c(t)$; $i \neq j$ corresponding to interval (k-1). The resulting signal is correlated with the symbol estimate $\ddot{b}_{k-j}$ to generate a noisy estimate $h_j^{c'}(t)$ of the chip $h_j^c(t)$.

$$h_j^{c'}(t) = [r_k(t) - \sum_{\substack{i=0 \\ i\neq j}}^{n} \ddot{b}_{k-i}\, \ddot{h}_i^c(t)]\ddot{b}_{k-j} \qquad j = 0,1,2, \ldots n.$$

$$= h_j^c(t) + e_{jk}(t)$$

Where the noise term $e_{jk}(t)$ has components due to additive noise $n_k(t)$ and the remanant ISI. The $e_{jk}(t)$ can be considered zero mean uncorrelated random variables with variace ε_j^2.

The circulating filter then operates upon $h_j^{c'}(t)$ to generate the signal $z_j(t)$ from which an essentially noiseless

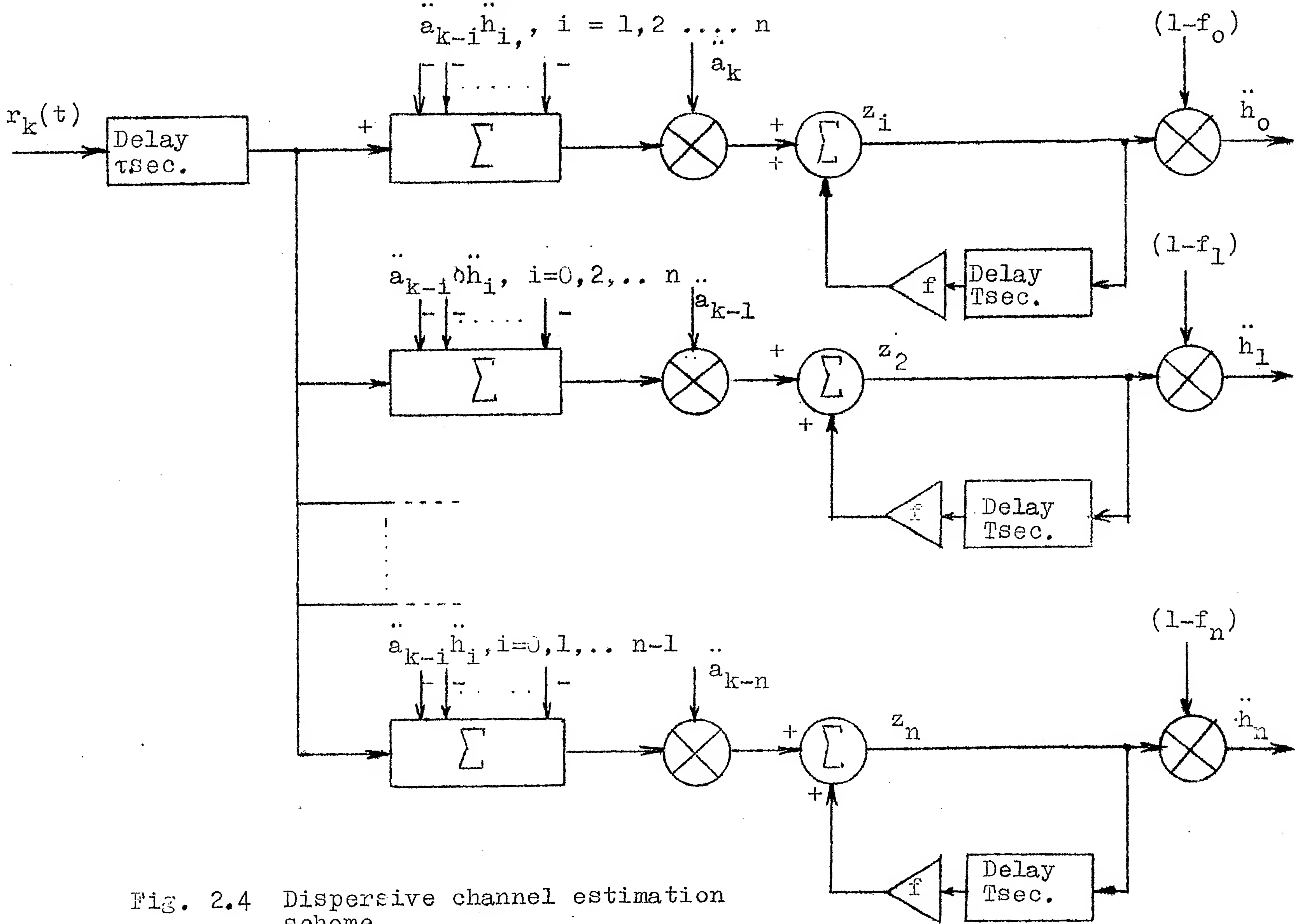


Fig. 2.4 Dispersive channel estimation scheme

estimate $\ddot{h}_j^c(t)$ of $h_j^c(t)$ is obtained.

$$z_j(t) = \frac{h_j^c(t)}{1 - f_j} + y_j(t)$$

where

$$y_j(t) = e_{jo} + f e_{j1} + f^2 e_{j2} + \ldots..$$

with a variance of φ_j^2

So that

$$\ddot{h}_j^c(t) = (1 - f_j)\, z_j(t)$$

$$= h_j^c(t) + (1-f_j)\, y_j(t) \quad j = 0,1, \ldots.\ n.$$

The resulting noise suppression is by a factor

$$\frac{(1 - f_j)^2 \quad \varphi_j^2}{\varepsilon_j^2}$$

The noise suppressing property of the circulating filter in the presence of ISI is considered next. We consider a case where the ISI is due to past two input symbols. Three circulating filters are used to estimate three segments $h_i^c(t)$; $i = 0,1,$ and 2, of the channel pulse response, $h^c(t)$. The system can be represented in matrix form as

$$\begin{bmatrix} \ddot{x}_o(k) \\ \ddot{x}_1(k) \\ \ddot{x}_2(k) \end{bmatrix} = \begin{bmatrix} f_o & -f'_1\ddot{b}_k\ddot{b}_{k-1} & -f'_2 b_k b_{k-2} \\ -f'_o\ddot{b}_k\ddot{b}_{k-1} & f_1 & -f'_2\ddot{b}_{k-1}\ddot{b}_{k-2} \\ -f'_o\ddot{b}_k\ddot{b}_{k-2} & -f'_1 b_{k-1} b_{k-2} & f_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_o(k-1) \\ \ddot{x}_1(k-1) \\ \ddot{x}_2(k-1) \end{bmatrix} + \begin{bmatrix} \ddot{b}_k \\ \ddot{b}_{k-1} \\ \ddot{b}_{k-2} \end{bmatrix} [r_k]$$

where $r_k = s_k + n_k$

f_i = feedback gain of the ith circulating filter $i = 0,1,2$

$f'_i = 1 - f_i \qquad i = 0,1,2$

$\ddot{x}_i(k)$ = filter output at time k; $i = 0,1,2.$

Or in matrix notation form -

$$\ddot{X}(k) = F\ \ddot{X}(k-1) + \ddot{B}\ r_k.$$

In case of no noise

$$\bar{X}(k) = \bar{F}\ \bar{X}(k-1) + \ddot{B}\ s_k$$

We define error matrix as

$$\ddot{X}_\varepsilon(k) \doteq \bar{X}(k) - \ddot{X}(k)$$

where $\quad x_{\varepsilon_i}(k) = x_i(k) - \ddot{x}_i(k) \quad , \quad i = 0,1,2.$

So that

$$X_\varepsilon(k) = F\ X_\varepsilon(k-1) + n_k\ \ddot{B}$$

We define error variance matrix as

$$\bar{P}(k) \doteq E[\bar{X}_\varepsilon(k)\ \bar{X}_\varepsilon^T(k)]$$

Substituting for $\bar{X}_\varepsilon(k)$ and $\bar{X}_\varepsilon^T(k)$

$$\bar{P}(k) = E\ [\ [\ \bar{F}\bar{X}_\varepsilon(k-1) + n_k\ \bar{B}]\ [\bar{F}\ \bar{X}_\varepsilon(k-1) + n_k\ \bar{B}]^T]$$

As source and noise statistics are independent

$$P(k) = E[F\ X_\varepsilon(k-1)\quad \bar{X}_\varepsilon^T(k-1)F^T] + \sigma^2\ B\ B^T$$

where σ^2 = noise variance.

$$= \bar{F}\ \bar{P}(k-1)\ \bar{F}^T + \sigma^2\ \bar{B}\ B^T$$

This after matrix multiplication and noise normalization and using the properties that the matrix $P(k)$ is symmetric and $[\ddot{b}_k]^2 = 1$ for all k can be written in the form of (1).

We assume that the steady state solution to the above system of equations is of the form

$$P_{ij} = \frac{1}{(1 - f_i)\ (f_o + f_1 + f_2 - 1)} \qquad i=j \quad i,j = 0,1,2$$

$$= 0 \qquad\qquad i \neq j \qquad\qquad ..\ (2)$$

That P_{ij} of above form is really a solution to (1) is seen by substituting (2) in (1). Thus P_{oo} under steady state assumption is

$$
\begin{bmatrix} P_{00}(K) \\ P_{11}(K) \\ P_{22}(K) \\ P_{10}(K) \\ P_{20}(K) \\ P_{21}(K) \end{bmatrix}
=
\begin{bmatrix}
f_0^2 & f_1'^2 & f_2'^2 & -2f_0f_1'\ddot{b}_k\ddot{b}_{k-1} & -2f_0f_2'\ddot{b}_k\ddot{b}_{k-2} & 2f_1'f_2'\ddot{b}_{k-1}\ddot{b}_{k-2} \\
f_0'^2 & f_1^2 & f_2'^2 & -2f_1f_0'\dot{b}_k\ddot{b}_{k-1} & 2f_0f_2'\ddot{b}_k\ddot{b}_{k-2} & -2f_1f_2'\ddot{b}_{k-1}\ddot{b}_{k-2} \\
f_0'^2 & f_1'^2 & f_2^2 & +2f_0'f_1'\dot{b}_k\ddot{b}_{k-1} & -2f_2f_0'\dot{b}_k\ddot{b}_{k-2} & -2f_2f_1'\ddot{b}_{k-1}\dot{b}_{k-2} \\
-f_0f_0'\dot{b}_k\dot{b}_{k-1} & -f_1'f_1\ddot{b}_kb_{k-1} & f_2'^2\ddot{b}_k\ddot{b}_{k-1} & f_0f_1+f_0'f_1' & -f_2'(f_0-f_0')\dot{b}_{k-1}\ddot{b}_{k-2} & -f_2'(f_1-f_1')\ddot{b}_k\ddot{b}_{k-2} \\
-f_0f_0'\ddot{b}_k\ddot{b}_{k-2} & f_1'^2\dot{b}_k\dot{b}_{k-2} & -f_2f_2'\ddot{b}_k\ddot{b}_{k-2} & -f_1(f_0-f_0')\dot{b}_{k-1}\dot{b}_{k-2} & f_0f_2+f_0'f_2' & -f_1'(f_2-f_2')\ddot{b}_k\ddot{b}_{k-1} \\
f_1'^2\dot{b}_{k-1}\dot{b}_{k-2} & -f_1f_1'\ddot{b}_{k-1}\ddot{b}_{k-2} & -f_2f_2'\dot{b}_{k-1}\dot{b}_{k-2} & -f_0'(f_1-f_1')\ddot{b}_{k-1}\dot{b}_{k-2} & -f_0(f_2-f_2')\ddot{b}_k\ddot{b}_{k-2} & f_1f_2+f_1'f_2'
\end{bmatrix}
\begin{bmatrix} P_{00}(k-1) \\ P_{11}(k-1) \\ P_{22}(k-1) \\ P_{10}(k-1) \\ P_{20}(k-1) \\ P_{21}(k-1) \end{bmatrix}
+
\begin{bmatrix} 1 \\ 1 \\ 1 \\ \ddot{b}_k\ddot{b}_{k-1} \\ \ddot{b}_k\ddot{b}_{k-2} \\ \ddot{b}_{k-1}\ddot{b}_{k-2} \end{bmatrix}
\quad \text{- - - - - - - - - - - (1).}
$$

$$P_{oo}(k) = f_o^2 P_{oo}(k-1) + f_1'^2 P_{11}(k-1) + f_2'^2 P_{22}(k-1)$$

$$- 2f_o f_1' \ddot{b}_k \ddot{b}_{k-1} P_{10}(k-1) - 2f_o f_2' \ddot{b}_k \ddot{b}_{k-2} P_{20}(k-1)$$

$$+ 2f_1' f_2' \ddot{b}_{k-1} \ddot{b}_{k-2} P_{21}(k-1) + 1$$

or,

$$P_{oo}(k) = f_o^2 \frac{1}{(1-f_o)(f_o+f_1+f_2-1)} + \frac{(1-f_1)^2}{(1-f_1)(f_o+f_1+f_2-1)}$$

$$+ \frac{(1-f_2)^2}{(1-f_2)(f_o+f_1+f_2-1)} + 1$$

$$= \frac{1}{(1-f_o)(f_o+f_1+f_2-1)} [f_o^2 + (1-f_o)(1-f_1)$$

$$+ (1-f_o)(1-f_2) + (1-f_o)(f_o+f_1+f_2-1)]$$

$$= \frac{1}{(1-f_o)(f_o+f_1+f_2-1)}$$

$$= P_{oo}(k)$$

The error variance coefficients P_{ij} are measure of noise power ; since for $i = j$, $\varphi_j^2 = P_{jj}$. The noise suppression due to j^{th} circulating filter is by a factor

$$(1 - f_j)^2 \times \frac{1}{(1-f_j)(f_o+f_1+f_2-1)} \; ; \quad j = 0,1,2.$$

$$= \frac{1-f_j}{f_o+f_1+f_2-1}$$

The corresponding SNR improvement is

$$SNR_{imp} = 10 \log_{10} \frac{f_0+f_1+f_2-1}{1-f_j} \text{ dB} ; j = 0,1,2.$$

For $f_j = 0.9$ $j = 0,1,2$; the SNR improvement when there is ISI due to past two symbols is

$$SNR_{imp} = 10 \log_{10}[17] = 12.3 \text{ dB}.$$

A similar result when there is ISI due to past three symbols is

$$SNR_{imp} = 10 \log_{10} \left[\frac{f_0+f_1+f_2+f_3-2}{1-f_j}\right] \text{ dB} ; j = 0,1,2,3.$$

For $f_j = 0.9$; $j = 0,1,2,3$ the SNR improvement is 12.2 dB.

These results indicate that the penalty is only about 0.5 dB when ISI is due to past 2 or 3 symbols and still substantial SNR improvement of more than 12 dB is achieved by using circulating filters.

For most practical channels, especially for tropo-applications, the estimator must be capable of estimating any unknown channel and tracking any variations therein in a finite number of steps. That this stability criterion is satisfied by circulating filter estimators is demonstrated by considering filter equations with any arbitrary initial conditions $\ddot{X}_1(0)$

$$\ddot{\bar{X}}_1(1) = \bar{F}\,\ddot{\bar{X}}_1(0) + \ddot{\bar{B}}\ r_0$$

The system with correct initial conditions is

$$\ddot{\bar{X}}(1) = \bar{F}\,\ddot{\bar{X}}\,(0) + \ddot{\bar{B}}\,r_0$$

It is shown that the error matrix

$$\bar{Z}(k) \doteq \ddot{\bar{X}}\,(k) - \ddot{\bar{X}}_1(k)$$

tends to $\bar{0}$ as k becomes larger and larger. Detailed derivations will be given in some future work.

In our discussion we have assumed that error-free estimates $\ddot{b}_k$ are available. Since the estimation technique is decision - directed its performance will be affected by frequent decision errors. However, in practical situations the error event is very rare and the estimator performance is only negligibly, if at all, degraded. Since all practical receivers make decisions after a delay of τ sec. (decision delay), same delay will appear in estimates also. In all situations of interest the channel varies slowly as compared to data rate and the system is not significantly affected by using delayed estimates.

2.5 Simulation Results

The channel estimation scheme described in this chapter was simulated on a digital computer (IBM-7044). The channel

response estimated had ISI due to past three symbols and was corrupted by white Gaussian noise. The feedback path-gains of all circulating filters were 0.9 and four channel samples per baud were estimated. The performance of the estimator was studied in the following contexts -

a) SNR Improvement

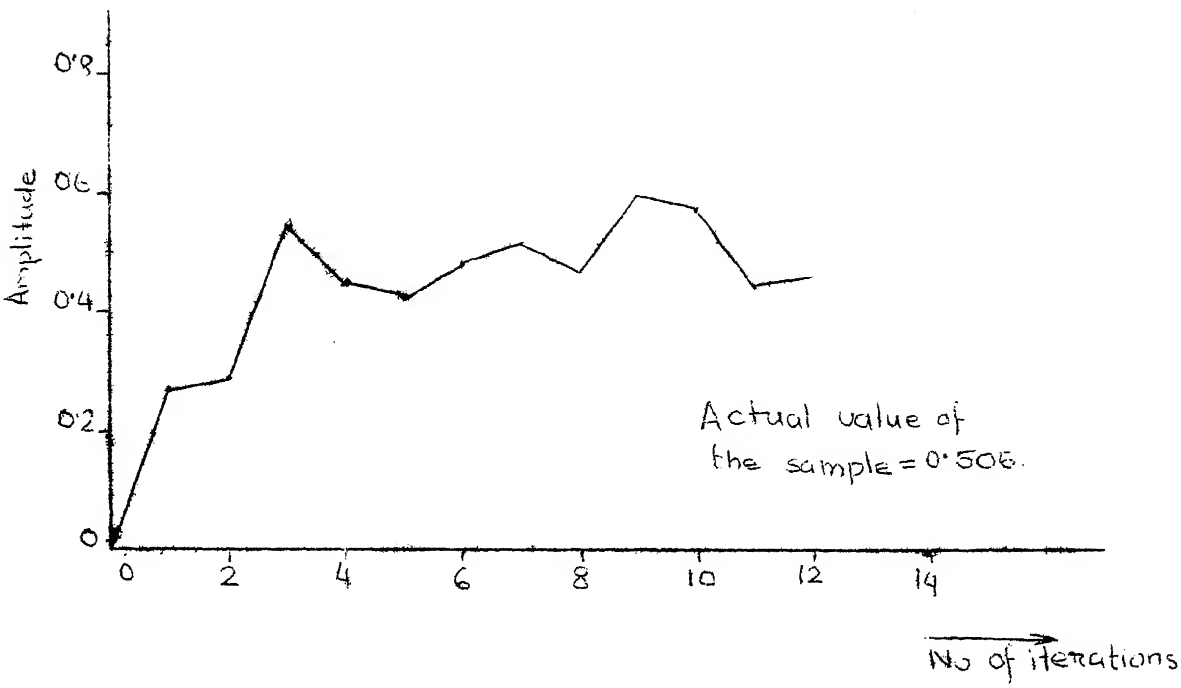
The SNR improvement due to circulating filters was about 12 dB for all the four circulating filters. This closely follows theoretical results.

b) Convergence

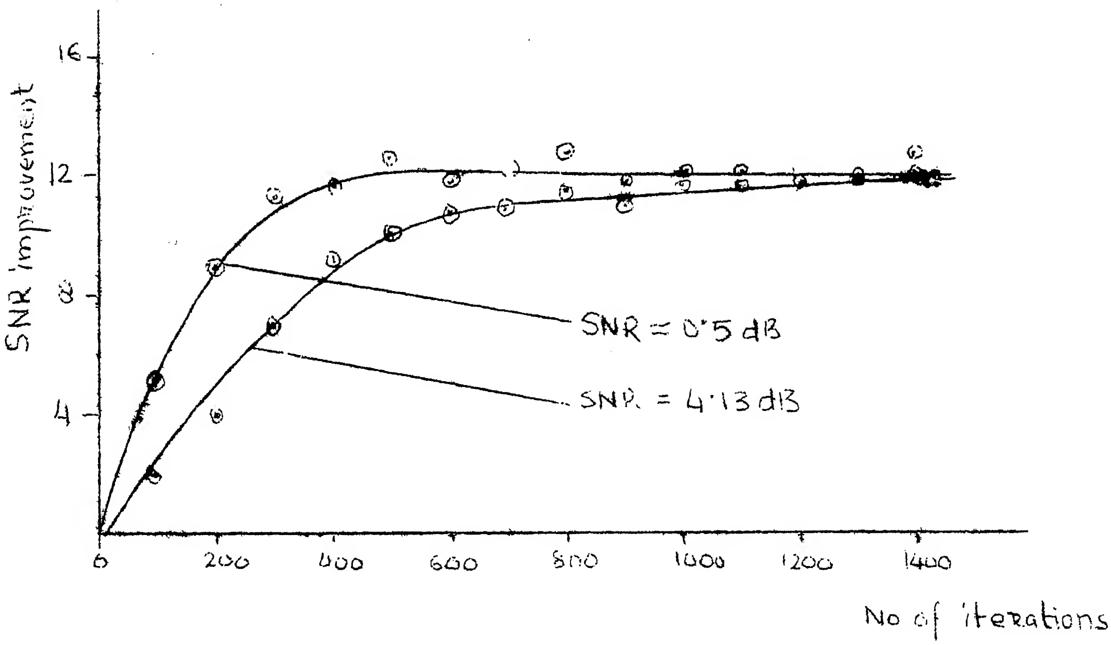
The estimates were found to converge to correct values within 50 data bits when the starting values were zero. It took somewhat more time in attaining the 12 dB SNR improvement value. For trailing segments (segments with lower SNR's) the 12 dB value was reached earlier than for segments with higher SNR's.

c) Decision Errors

The effect of erroneous decisions on channel estimates was studied by deliberately introducing bursts of decision errors. It was observed that bursts of upto 5 errors do not significantly affect the channel estimates. But with a burst of 10 errors the channel acquisition was completely lost.



Estimate build up by a circulating filter



SNR improvement in a Circulating Filter

d) Effects of Time-jitter

Time-jitters of T/4 and T/2 were introduced in the received signal and the effect on channel estimates was studied. With +T/4 jitter all channel samples are correctly estimated except for every fourth sample which now depends on data sequence. With -T/4 time jitter, first sample of all T sec. segments of channel response are erroneous. With time jitters of T/2 sec only two out of four samples per T sec. are correctly estimated.

CHAPTER 3

DFE RECEIVER USING CIRCULATING FILTERS FOR TROPOSCATTER CHANNELS

3.1 Receivers for Troposcatter Channels

The fading dispersive character of troposcatter channels results in the received signal fading and its dispersion over several symbol durations. Fading reduces protection from thermal noise, whereas dispersion leads to intersymbol interference. The channel characteristics are in fact a major element to be evaluated in considering any communication system for troposcatter channels. In this context the channel estimator assumes greater significance in the receiver configurations.

The performance of digital communication systems over troposcatter channels is thus degraded predominantly by ISI and fading besides additive noise. The degrading effects of noise and fading can be reduced by increasing the transmitted signal power and by explicit diversity techniques. But this way the ISI degradations cannot be minimized since, for linear channels, the ISI is a function of channel dispersion only and is independent of signal to noise ratio. Special techniques, known as channel equalization, are to be incorporated in the receiver to combat ISI. The equalizer is made

adaptive to accommodate any changes in the channel characteristics. Thus, an adaptive equalizer is a special feature of receivers for tropochannel applications.

The channel equalization techniques can be broadly categorized as linear and nonlinear techniques. The linear techniques like matched filtering followed by transversal filter are straightforward and economic but their performance is markedly inferior to nonlinear techniques like decision feedback equalization [GBS-71] and Viterbi algorithm [Vit-71]. Especially, the Viterbi algorithm technique has been shown to be optimum ML estimator [For-72, Kob-71].

Of the various equalization techniques proposed for troposcatter channels, the decision feedback equalization and Viterbi algorithm have assumed greater importance; the first one because of its practicability and fairly good performance and second one because of its optimality. Though VA receiver is optimum, there are certain difficulties in its hardware implementation like realization of a whitened matched filter, increasing complexity for larger dispersions of the channels and speed of operation.

In the present chapter we study the performance of the DFE receivers using circulating filters for tropochannel applications and the study of the performance of circulating filters in a VA receiver for unknown dispersive channels is

done in the next chapter.

3.2 DFE Receiver

The decision feedback equalization technique was first proposed by Austin [Aus-67]. The optimum receiver derived by him is very complex and cannot be implemented because it requires correlation of the output of an infinite length transversal filter with all possible data sequences and also large number of exponentiators. Practical versions were proposed in subsequent papers by Brady [Bra-70], Monsen [Mon-71, Mon-73, MR-73, Mon-74], George et al [GBS-71] and others. Brady and Monsen have considered DFE receivers for troposcatter channels. Brady developed a system which is MMSE processor and uses an adaption algorithm based on steepest descent method as given by Widrow [Wid-66]. The receiver structure developed by Monsen is capable of handling data at higher rates and adapts to channel variations without any explicit knowledge about the channel impulse response.

The decision feedback equalizer works on the principle that the receiver decisions can be used in eliminating past symbol interference. The assumption involved is that the receiver decisions are essentially correct. This is a valid assumption for low bit error probabilities of the order of (10^{-5} or 10^{-6}) encountered in practical systems.

A receiver scheme using decision feedback equalization is shown in Fig. 3.1. The receiver decisions are passed through the feedback filter and its tap gains are adjusted to minimize the past digit interference. The remanant ISI due to the future symbols is minimized in the forward filter. The forward filter is also in the form of a tapped delay line and its tap gains are adjusted to minimise future symbol interference.

The received signal $r(t)$ is passed through a filter $h(-t)$ matched to the channel $h(t)$ and the output is sampled to give the signal

$$y_k = \int_{-\infty}^{\infty} r(t)\ h(t-kT)dt$$

The forward filter is characterized by tap gains c_i; $i = 0,1, \ldots\ldots N$ and the feedback filter by tap gains d_j; $j = 1,2, \ldots\ldots M$.

$$c(t) \doteq \sum_{i=0}^{N} c_i\ \delta(t-iT)$$

$$d(t) \doteq \sum_{j=1}^{M} d_j\ \delta(t-jT)$$

Then the equalizer output is

$$b'_k = \sum_{i=0}^{N} c_i\ y_{k+i} - \sum_{j=1}^{M} d_i\ \ddot{b}_{k-j}$$

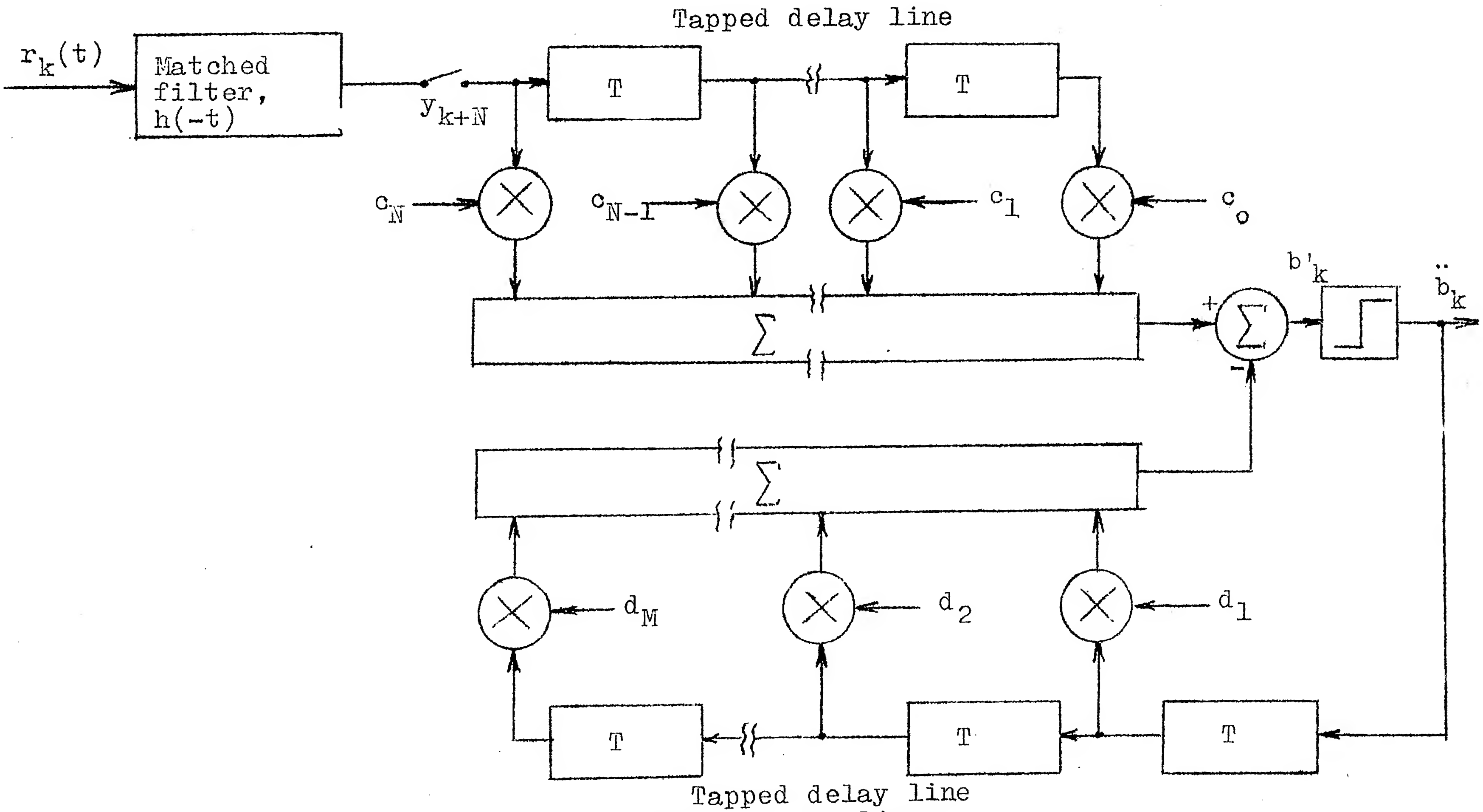


Fig. 3.1 Decision feedback equalizer

The equalizer output is converted to a decision $\ddot{b}_k$ with the help of a clipping circuit with zero bias. In other words, the equalizer output which forms sufficient statistics for making decision about b_k is checked against a zero threshold to give decision $\ddot{b}_k$

$$b'_k \underset{-1}{\overset{1}{\gtrless}} 0$$

The tap gains must be adjusted to minimize the probability that $\ddot{b}_k \neq b_k$. However, in the absence of analytical expressions for error probability in terms of tap gains, direct minimization of error probability is not possible. The tap gains are then selected to minimize the output mean square error, $E[e_k^2]$ where

$$e_k \doteq b'_k - b_k$$

The problem of adaptively adjusting the tap gains in MMSE sense is considered by George et al [GBS-71] and Monsen [Mon-71, Mon-74]. Monsen combined the matched filter and the forward TDL into a single forward filter $\gamma(-t)$. The tap weights, c_i, are then adjusted to minimize the effects of additive noise and future symbol interference. The forward filter is tapped at intervals on the order of the bandwidth reciprocal because it is approximating a matched filter as part of its response. For PSK systems the

tap spacing is T/2. The forward filter is then given by

$$\gamma(t) = \sum_{i=0}^{N} c_i \ \delta(t - iT/2)$$

This T/2 tap spacing results in some performance degradation due to inherent time quantization. Errors in time synchronization result in further degradation. Accurate synthesis of the forward filter in Monsen's structure is in fact not simple.

Monsen [Mon-74] has by way of simulation shown that the DFE receiver for tropochannels performs about 5 dB worse than the optimum one shot receiver. The degradation is attributed to reasons like only partial ISI cancellation, use of a finite number of taps and non-optimum sampling time locations. The last two factors result in inefficient realization of the forward filter. It has been pointed out in [Mon-74] that the remanent ISI penalty is not more than 1 dB. Then the major contribution of about 4 dB to the performance loss is from inefficient synthesis of the forward filter.

In Monsen's scheme [Mon-71, Mon-74] a separate algorithm is used for the adaption of the equalizer to the channel variations. The adaption is also possible using the channel impulse response estimates required for matched

filtering of the received signals and a separate of adaption algorithm is not needed.

3.3 DFE Receiver using Circulating Filters for Tropochannels

The Monsen's scheme of DFE receiver for troposcatter channels [Mon-71, Mon-74], as mentioned in the preceeding section, suffers from a performance loss due to inefficient forward filtering. The degradation is substantial and it is desirable to use a technique in which the forward filter is either better synthesized or it is totally done away by using some special property of the channel. In this section a receiver structure is considered which does not use a forward filter and utilizes channel estimates obtained by using circulating filter estimator of chapter 2, for both matched filtering and decision feedback equalization.

In deriving the receiver structure advantage is taken of the asymmetric nature of the power impulse response of the troposcatter channels. The delay power spectra of typical tropo-links are shown in Fig. 3.2. Fig. 3.3 depicts the received signal energy as a function of time. These plots show that the delay power spectrum is highly skewed and almost all signal energy is received in the first few symbol intervals. As much as 80 percent or even more of the signal energy of a pulse transmitted over a tropochannel is received in the first T sec.

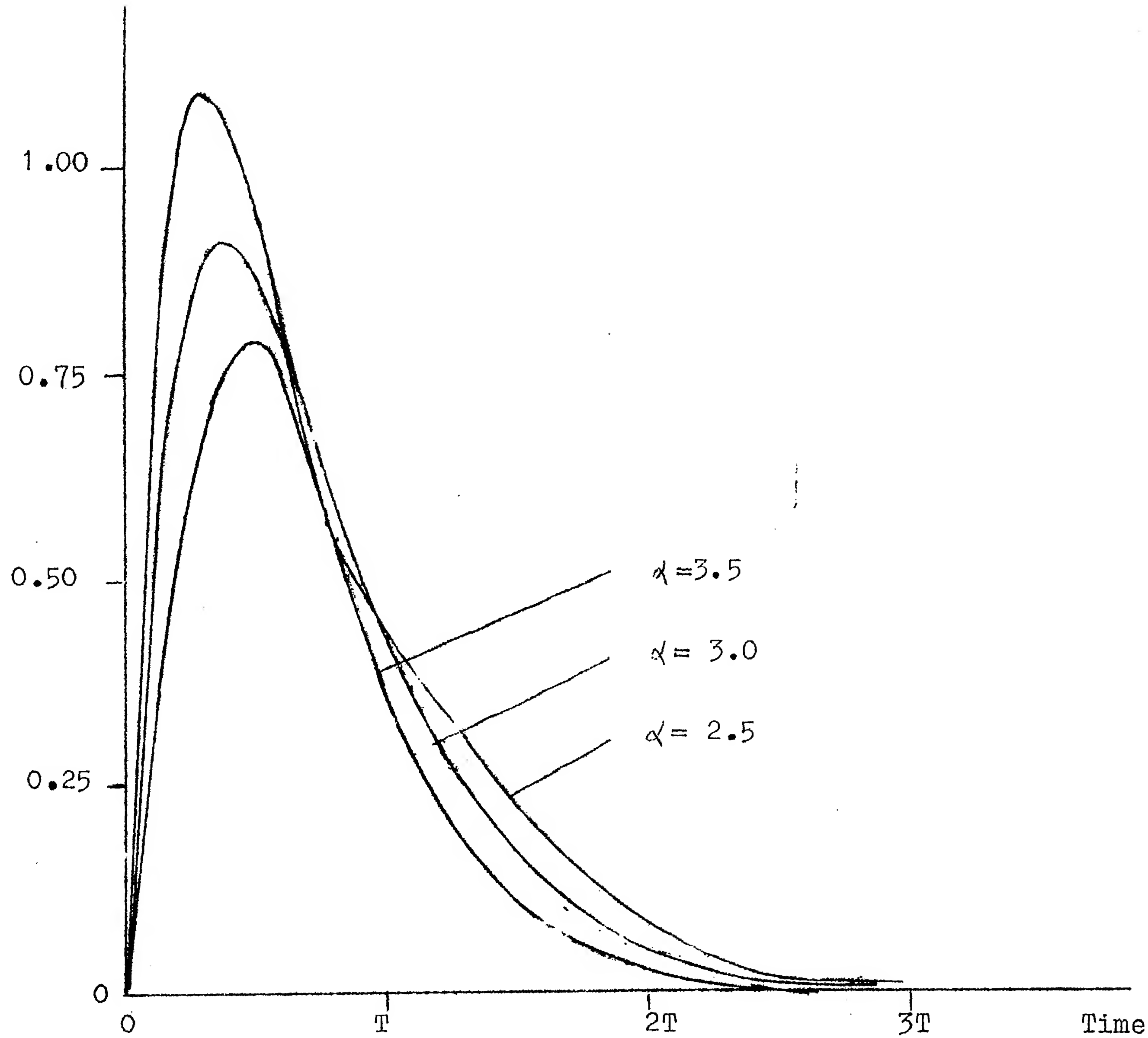


Figure 3.2: Delay Power Spectra of a Troposcatter Channel.

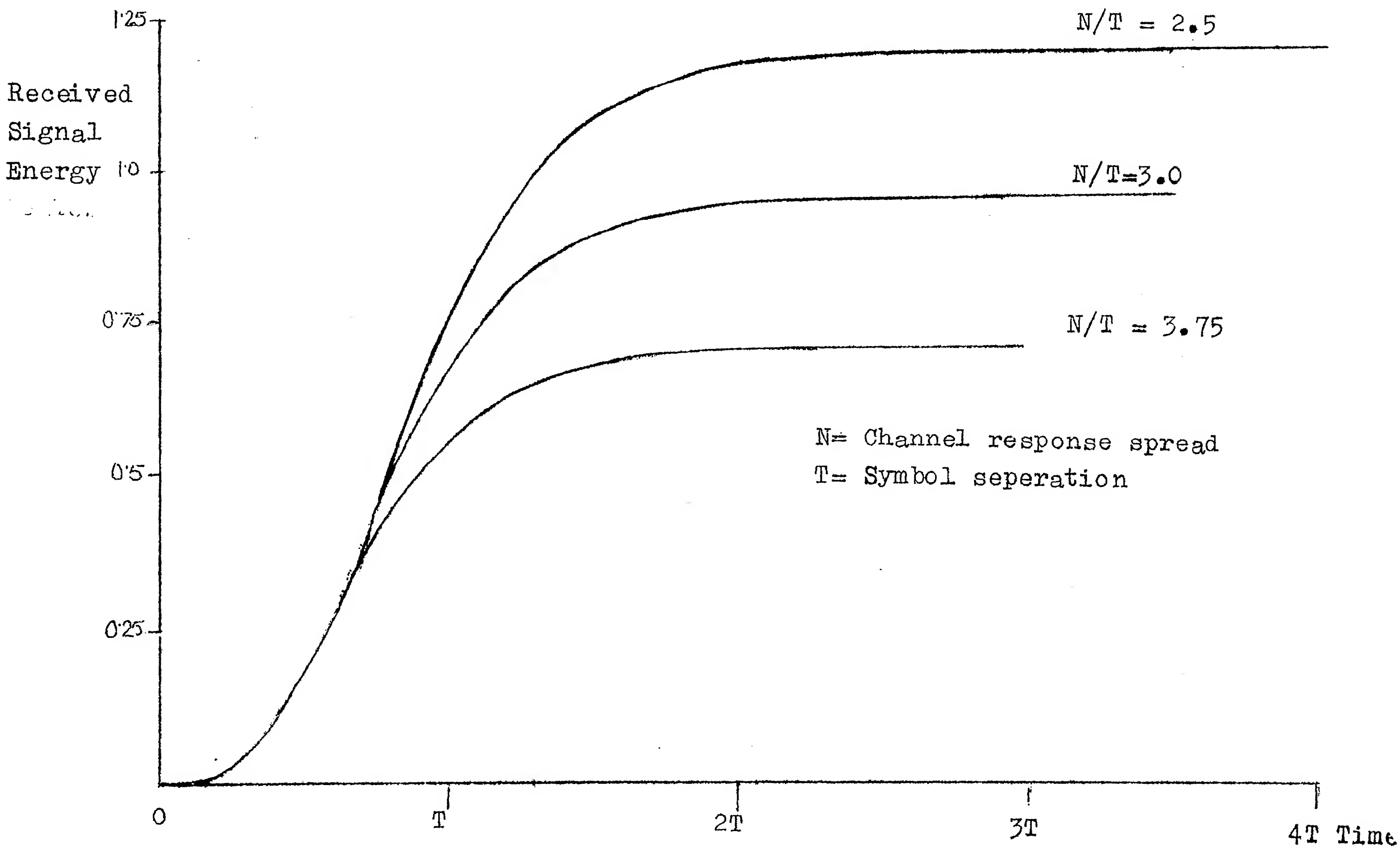


Figure 3.3: Received Signal Energy over a Troposcatter Channel.

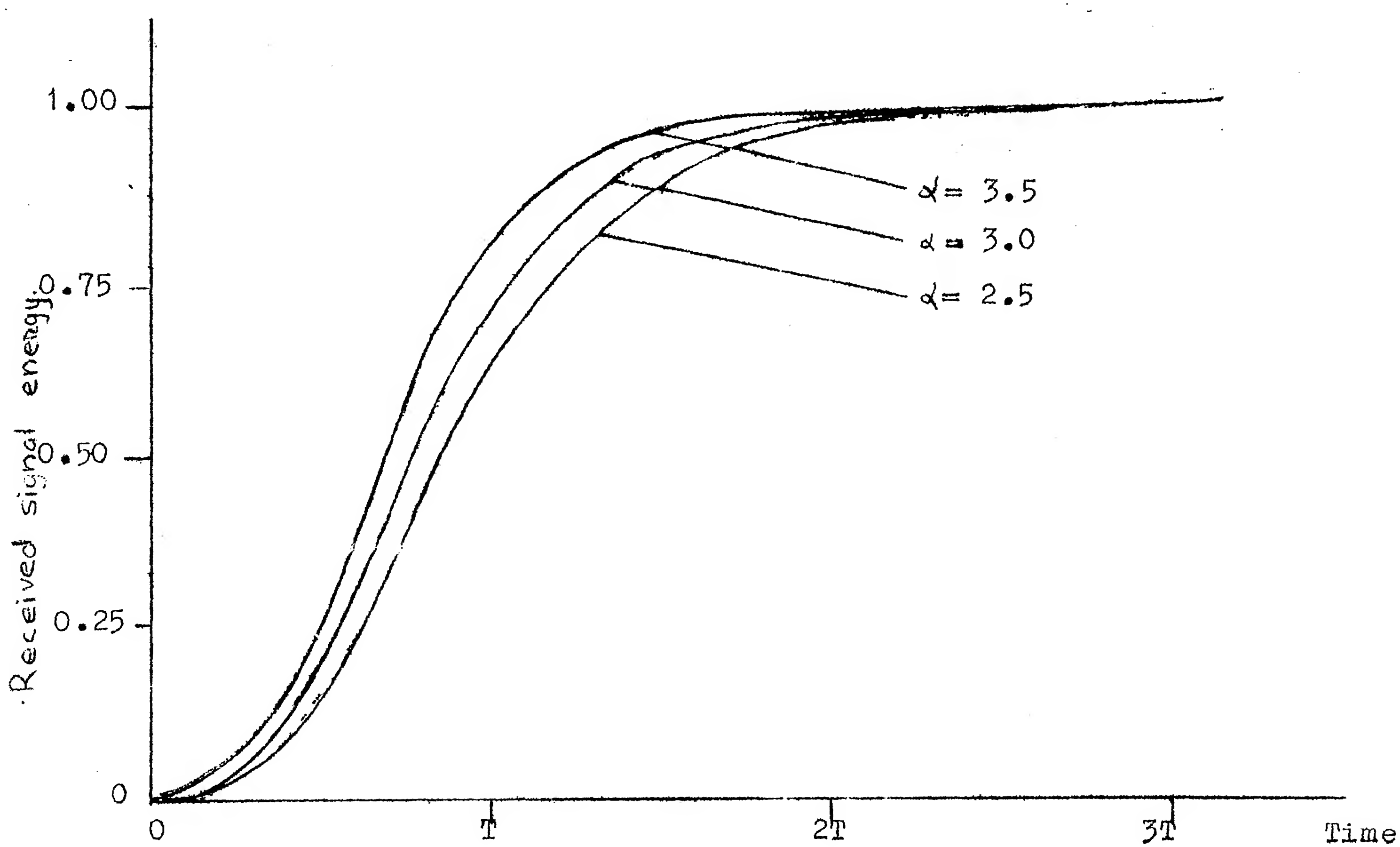


Figure 3.3A: Received Signal Energy as a function of α.

Then, if the decision about the transmitted symbol in the k^{th} interval is taken at the end of the k^{th} interval, there will be no contribution due to the symbol transmitted in $(k+1)^{th}$ interval but sufficient signal energy would be available for decisions. Thus by deciding about the transmitted bit at the end of T sec, the symbol duration, the future symbol interference is eliminated. The equalizer has to overcome the past symbol interference only for which the feedback filter alone is sufficient. We do lose some signal energy in making decisions at the end of T sec. but the resulting degradation is small as compared to that incurred with late decisions when a forward filter is a must.

3.3.1 Receiver Structure

A schematic representation of the adaptive DFE receiver with the above mentioned modifications is depicted in Fig. 3.4. The receiver has four blocks: a channel estimator, a decision feedback equalizer, a matched filter and a threshold detector. The received signal is delayed by T sec. in the channel estimator and then the information about the channel response is extracted from it using circulating filters. The channel estimates $\ddot{h}_j^c$ are available after a delay of T sec. but for normal data rates of information transmission over slowly fading tropo channels, this delay is immaterial. The

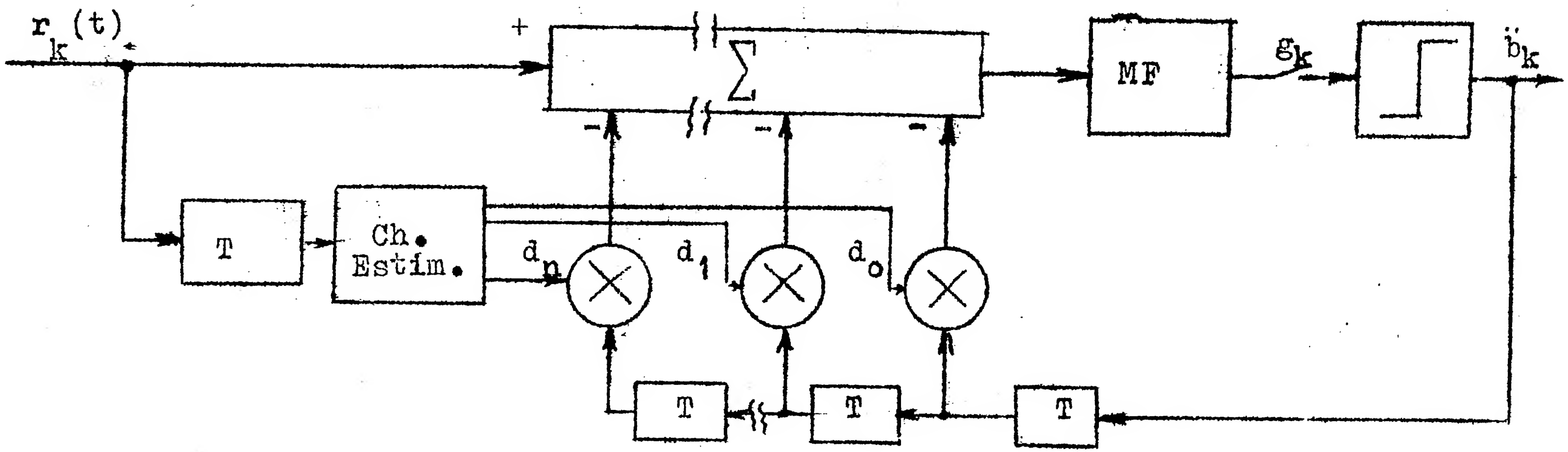


Figure 3.4: Adaptive DFE Receiver for troposcatter Channels.

feedback filter is in the form of a tapped delay line with n taps. The tap gains are set equal to the chip estimates of the composite channel impulse response. The receiver decisions are passed through the feedback filter. At time k the filter output is

$$u_k(t) = \sum_{j=1}^{n} \ddot{b}_{k-j} \, \ddot{h}_j^c(t)$$

which corresponds to the past symbol interference in the received signal

$$v_k(t) = \sum_{j=1}^{n} b_{k-j} \, h_j^c(t)$$

The received signal r_k is

$$r_k(t) = \sum_{j=0}^{n} b_{k-j} \, h_j^c(t) + n_k(t)$$

$$= b_k \, h_o^c(t) + v(t) + n_k(t)$$

Then, the output of the equalizer is

$$b_k'(t) = b_k \, h_o^c(t) + \sum_{j=1}^{n} [b_{k-j} \, h_j^c(t) - \ddot{b}_{k-j} \, \ddot{h}_j^c(t)] + n_k(t)$$

$$= b_k \, h_o^c(t) + w(t) + n_k(t)$$

where the remanant ISI w(t) is

$$w(t) = v(t) - u(t)$$

It will be quite small when receiver decisions are correct and the channel estimates are fairly accurate. The equalizer output is matched filtered by correlating it with the locally generated chip $\ddot{h}_o^c$. The sampled output g_k of the matched filter at the end of kth interval, is sufficient statistics for making the decision about the transmitted symbol b_k and a decision $\ddot{b}_k$ as to b_k is taken by the hypothesis test.

$$g_k \underset{-1}{\overset{1}{\gtrless}} 0 \qquad \text{where} \quad g_k = \int_{-\infty}^{\infty} b_k'(t)\, \ddot{h}_c(t).$$

3.3.2 Diversity Reception

Diversity reception is also incorporated in the system. Maximal ratio combining is used in combining the received signals on different branches. For Lth order diversity the received signals on L branches in the kth interval are $[r_k^i(t)\ ;\ i = 1,2, \ldots\ L]$. Each of the received signals is acted upon by seperate channel estimators, equalizers and matched filters matched to corresponding channels. The sampled MF outputs at the end of kth interval are $[g_k^i,\ i = 1,2, \ldots\ L]$. The sufficient statistics as to the transmitted symbol is then

$$G_k = \sum_{i=1}^{L} g_k^i$$

and decisions, $\ddot{b}_k$, are made according to the rule

$$G_k \underset{-1}{\overset{1}{\gtrless}} 0 .$$

3.4 Performance of the DFE Receiver for Troposcatter Channels

The performance of adaptive DFE receiver is limited by a self noise due to the adaption process and by intersymbol interference. The first effect can be made reasonably small if the channel is slow fading, i.e. if channel variations occur at a rate much less than the data rate. Under this condition the behaviour is limited by ISI. In addition, error-propagation also affects the performance. Due to the nonlinear receiver structure, it is not easy to get closed form expressions for probability of error. The system performance is then often expressed either in terms of performance bounds or simulation results.

The performance bounds are obtained in terms of the performance of a one-shot receiver. The one-shot receiver is defined as the receiver that detects only a single digit. This idealization ignores ISI and provides an error probability bound for sequential reception of digits. For no diversity case it is simply a matched filter receiver. L

dimensional extension of the MF receiver gives one-shot receiver for Lth order diversity reception.

The probability of digit error for one shot receiver is given by

$$p(\gamma) = \tfrac{1}{2}\ \mathrm{erfc}\ (\sqrt{\gamma}) \qquad (1)$$

where γ is signal energy to noise power ratio E/N_o. N_o is single sided spectral density of the additive Gaussian noise process. The received signal energy is given by

$$E = \int_{-\infty}^{\infty} |H^c(f)|^2\, df$$

where $H^c(f)$ is fourier transform of the composite channel impulse response $h^c(t)$.

Since the channel is random, γ is a random variable. An important performance measure is the probability of error averaged over γ. For a given average E/N_o average probability of error is determined by integrating (1) over the probability density function for γ. This density can be found by considering a sampled representation of transfer functions $H^c(f)$. This is done in troposcatter channel context by Monsen [Mon-73] and an expression for average probability of error $P(\mu)$, for an independent branch Lth-order diversity system with an average $E/N_o = \mu(>1)$ is obtained in the form of a convergent series

$$P(\mu) = \frac{\sqrt{\mu}}{2\pi} \sum_{i=L}^{\infty} \frac{\Gamma(i+0.5)}{(1+\mu)^{i+0.5} i!}$$

3.4.1 Error Propagation Effects

In the receiver configuration described in the previous section, the receiver estimates of the transmitted bits are used in the removal of the intersymbol interference. These decisions are also used in the channel response estimation. If there is any erroneous decision made, it will be propagated back by the feedback filter of the equalizer and the intersymbol interference will be only partially cancelled. Under such circumstances, there is a higher probability of receiver making further errors in its decisions. Error propagation in channel estimator affects the channel estimates. Incorrect channel estimates result in incomplete ISI cancellation and inadequate matched filtering thereby further increase the error paprobability. This, any error in decision by the receiver increases the chances of further errors in near future and the decision errors tend to occur in bursts. However, for DFE receivers, it has been pointed out in [GBS-71] that the receiver is able to recover from error conditions and long bursts of errors occur only rarely.

The performance of the DFE receiver under error propagation conditions has been studied by Austin [Aus-67], Monsen [MR-73 and Mon-74] and Cantoni and Butler [CB-76] using two different approaches. In the first approach adopted by Austin and Monsen the error propagation is modelled as a discrete Markov chain process and bounds on mean recovery time and bit error probability have been obtained. But it has been pointed out [CB-76] that the statistics of the noise are critical to the development of a Markov model and that the method is suitable for error probability calculations for short feedback paths only. The approach adopted by Cantoni and Butler is based on the theory of success runs. Using this technique, they have derived bounds for the probability of recovery from error in a given number of steps and for mean recovery time which do not depend on the statistics of the noise and particular selection of the channel.

3.5 Simulation Results

A digital communication system for dispersive fading channels using the DFE receiver described in this chapter was simulated on digital computing systems - IBM - 7044 and DEC-10. Details of simulation programs are given in Appendix I. The simulation results are given below.

a) Average Probability of Error - No Diversity

The average probability of error characteristic of the

system as a function of average signal to noise power ratio is depicted in the plot of Fig. 3.5. Also shown is the average error rate performance of an optimum receiver for a non-dispersive flat fading channel.

The average prob. of error for the system simulated was obtained by averaging the results over one million data bits and 20 different realizations of the channel. These results are tabulated in Table 1.

b) Average Probability of Error-Quad Diversity

A diversity reception of the order of four was considered and results for 20 different realizations of the channel over one million data symbols were obtained for four SNR values. Errors were obtained only for the lowest SNR value considered; i.e. 6 dB when 3 errors occured. For higher SNR's no decision errors were observed. The results are given in Table 2.

c) Error Propagation

The error propagation effects on the performance of the receiver were studied by deliberately introducing bursts of errors in the system. The results are given in Table 3. Bursts of upto 5 errors resulted in no extra errors. Bursts of 6 error resulted in 2 additional decision errors, burst

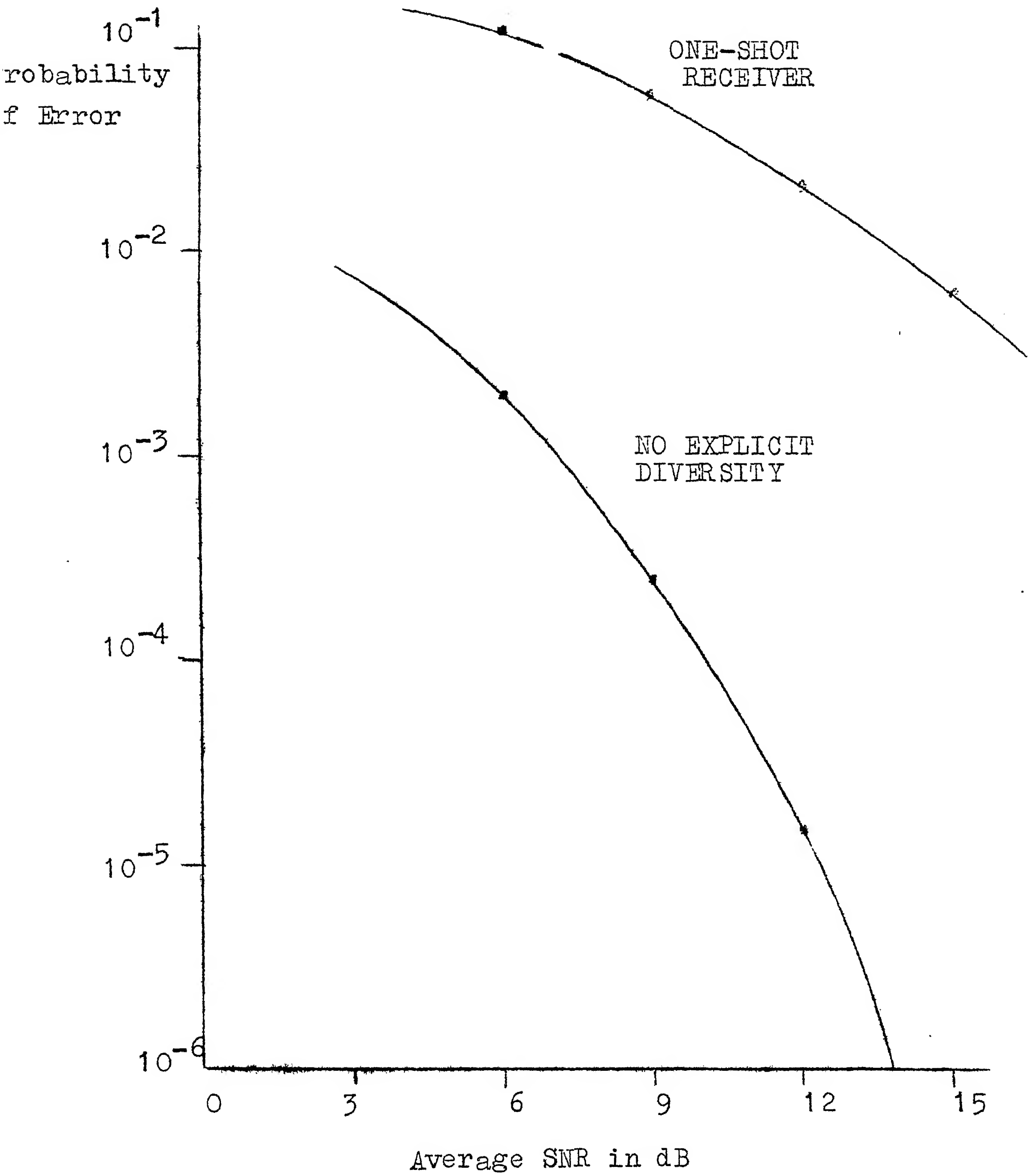


Figure 3.5: DFE Receiver Performance

Table 1 Probability of error performance - No diversity

Channel Realizations	1	2	3	4	5	6	7	8	9	10	
SNR = 15 dB	0	0	0	0	0	0	0	0	0	0	
SNR = 12 dB	0	0	0	0	157	1	135	1	0	0	
SNR = 9 dB	83	4	0	1	3381	102	3194	299	3	0	
SNR = 6 dB	2454	539	12	248	19759	2879	18854	4738	565	0	
Channel Realizations	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**Error rate**
SNR = 15 dB	0	0	0	0	0	0	0	0	0	0	0
SNR = 12 dB	0	0	0	0	0	7	1	0	0	0	1.51×10^{-5}
SNR = 9 dB	4	5	0	0	12	102	17	35	0	0	3621×10^{-5}
SNR = 6 dB	335	644	32	52	228	2861	466	2070	2	13	283.75×10^{-5}

Normalized standard deviation relative to mean

SNR (dB)	NSD
15	0
12	2.90
9	2.70
6	1.99

Table 2
Probability of error performance - Quad diversity

Channel realizations	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Error rate
SNR = 15 dB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SNR = 12 dB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SNR = 9 dB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SNR = 6 dB	0	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.5×10^{-7}

Table 3 Error propagation effects

Burst length	No. of errors*
1	0
2	0
3	0
4	0
5	0
6	2
7	5
8	68

*Number of errors in 100 bits after burst occurance. The results are with quad-diversity and SNR = 3 dB.

Table 4 Time-jitter effects

Time-jitter (in units of T)	No. of errors in 10^4 bits*
0	0
+1/4	0
-1/	1
+1/2	374
-1/2	261

* Average SNR = 3 dB Quad diversity

of 7 errors resulted in 5 additional errors and larger bursts resulted in complete loss of data acquisition. The average SNR at the receiver input was 3 dB.

d) Time-jitter

The receiver performance was studied under the effects of time-jitter by introducing time-jitters of $\pm$ T/4 and $\pm$ T/2 in the received signal. The results are given in Table 4. The results are for an SNR of 3 dB.

CHAPTER 4

VITERBI ALGORITHM RECEIVER USING CIRCULATING FILTERS

In most of the practical data communication systems symbol-by-symbol decisions are made mainly due to the simplicity of the operations involved. However, since better performance can be achieved with sequential decoding it is desirable that the receiver decisions be based on the entire received sequence. Unfortunately, the straightforward statistically optimum receiver has a complexity exponentially increasing with data length [Hel-60] and is not practical. The Viterbi algorithm [Vit-67] provides a way out and gives maximum likelihood estimates of the transmitted sequence with greatly reduced efforts [For-72]. The algorithm is based on the fact that practical channels can be represented as finite state machines. Since a finite state machine has finite number of state-transitions only these many transitions are to be considered by the receiver. Thus the Viterbi algorithm is an optimum receiver with fixed complexity which bases its decisions on the entire received sequence.

The algorithm was first proposed by Viterbi in 1967 [Vit-67] for decoding convolutional codes. Later on the method was shown to be a variation of the dynamic programming

problem by Omura [Omu-69] and to be a maximum likelihood estimator by Forney [For-72] and Kobayashi [Kob-71]. The technique has been studied in detail by Viterbi [Vit-71] and Forney [For-73] in the context of convolutional codes. In another paper [For-73], Forney presents a general discussion on the application of the algorithm to decision problems. In more recent papers like those of Mager and Proakis [MP-73], Qureshi and Newhall [QN-73] and Mager and Falconer [MF-76A, MF-76B] attention is directed towards increasing the practicability of the VA receivers by presenting it with a limited memory channel.

In this chapter a Viterbi Algorithm receiver using circulating filters for unknown dispersive channels which may be fading is considered. In section 1 a VA receiver structure using whitened matched filtering is considered. In section 2 the Viterbi algorithm is described. Its performance is considered in section 3 and in section 4 a receiver structure for tropochannel applications using circulating filters is described. In section 5 some simulation results for tropo and telephone channels are given.

4.1 VA Receiver Structure

A receiving scheme employing the concepts of Viterbi decoding is shown in Fig. 4.1. It consists of a whitened matched filter, a symbol rate sampler and the Viterbi decoder.

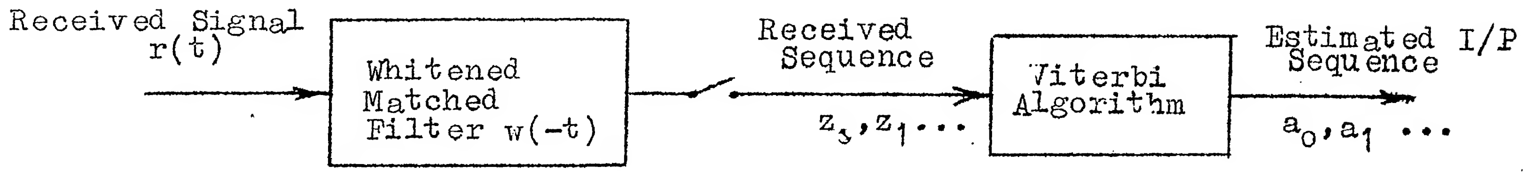


Figure 4.1: Maximum - Likelihood Sequence estimator.

For the case of finite impulse response channel and white noise the output of a filter $h(-t)$ matched to the channel $h(t)$ forms sufficient statistics for the estimation of the input sequence [Van-68]. Thus the matched filter outputs

$$x_k = \int_{-\infty}^{\infty} r(t)\ h(t-kT)\ dt$$

where $r(t)$ is received signal, form sufficient statistics for the estimation of the input sequence a_k. Since the received signal $r(t)$ is

$$r(t) = \sum_{k'} a_{k'}\ h(t-k'T) + n(t)$$

x_k can be written as, after interchanging integration with summation

$$x_k = \sum_{k'} a_{k'} \int_{-\infty}^{\infty} h(t-kT)\ h(t-k'T)\ dt + \int_{-\infty}^{\infty} h(t)\ h(t-kT)\ dt$$

$$= \sum_{k'} a_{k'}\ R_{k-k'} + n'_k$$

where

$$R_{k-k'} = \int_{-\infty}^{\infty} h(t-k'T)\ h(t-kT)\ dt \qquad |k - k'| \leq n - 1$$

$$= 0 \qquad \text{Otherwise.}$$

is the pulse auto-correlation coefficient of $h(t)$ and there are $2n-1$ coefficients. Using the delay power series representations

$$x(D) \doteq x_0 + x_1 D + x_2 D^2 + \ldots\ldots$$

$$a(D) \doteq a_0 + a_1 D + a_2 D^2 + \ldots\ldots$$

$$R(D) \doteq R_{-(n-1)}\, D^{-(n-1)} + R_{-n+2}\, D^{-n+2} + \ldots + R_{n-2} D^{n-2} + R_{n-1} D^{n-1}$$

$$n'(D) \doteq n'_0 + n'_1 D + n'_2 D^2 + \ldots\ldots$$

We can write

$$x(D) = a(D)\, R(D) + n'(D)$$

Here $n'(D)$ is zero mean coloured Gaussian noise with auto-correlation function $\sigma^2\, R(D)$ where σ^2 is spectral density of $n(t)$. Due to the coloured noise, the filter output cannot be used in the Viterbi decoder and whitened matched filter which gives uncorrelated noise at its output is needed.

Since $R(D) = R(D^{-1})$, its roots occur in pairs of the form (β, β^{-1}). Collecting one root from each pair polynomials $f(D)$ and $f(D^{-1})$ are formed such that

$$R(D) = f(D)\, f(D^{-1}).$$

Then we can write the coloured noise as

$$n'(D) = n(D)\, f(D^{-1})$$

since $n'(D)$ then has variance $\sigma^2\, f(D^{-1})\, f(D) = \sigma^2\, R(D)$ and zero mean Gaussian process is completely specified by its variance.

Then,

$$x(D) = a(D)\ f(D)\ f(D^{-1}) + n(D)\ f(D^{-1})$$

or,

$$z(D) \doteq \frac{x(D)}{f(D^{-1})} = a(D)\ f(D) + n(D).$$

in which noise is white.

The sequence z(D) can be obtained by sampling the outputs of the cascade, termed whitened MF, of a MF h(-t) and a transversal filter $1/f(D^{-1})$. The whitened MF is realizable when $f(D^{-1})$ has no roots on or inside the unit circle.

That the sequence z(D) forms sufficient statistics for decisions about the sequence a(D) is clear from the fact that the sufficient statistics x(D) can be recovered by passing z(D) through the filter $f(D^{-1})$. The Viterbi decoder then operates upon the sequence z(D) to generate ML estimates of the sequence a(D).

4.2 The Viterbi Algorithm

The input to the Viterbi decoder is

$$\begin{aligned} z(D) &= a(D)\ f(D) + n(D) \\ &= y(D) + n(D) \end{aligned}$$

The decoder obtains ML estimate of the sequence a(D) by maximizing the log likelihood function

$$\Lambda' \;=' \; \ln p[z(D) / a(D)]$$

or equivalently

$$\Lambda = \ln p[z(D) / y(D)]$$

Since the noise samples are independent Gaussian distributed

$$\Lambda = \sum_{k} \ln p[z_k / y_k]$$

where

$$p[z_k/y_k] = \frac{1}{\sigma\sqrt{2\pi}} \exp[- \frac{(z_k - y_k)^2}{2\sigma^2}]$$

We define

$$\Lambda_k \doteq \sum_{k_1=0}^{k-1} \ln p[z_{k_1}/y_{k_1}]$$

and

$$\delta \Lambda_k \doteq \ln \; p[z_k/y_k]$$

so that

$$\Lambda_{k+1} = \Lambda_k + \delta \Lambda_k$$

The log likelihood function is thus represented as a sum of independent increments. The following description of the algorithm illustrates how the maximization of the likelihood function is recursively achieved in Viterbi decoding.

The Viterbi algorithm can be described either by finite state machine approach or by trellis diagram method. We follow the second approach where the lowpass equivalent filter h(t) of the channel is modelled as convolutional encoder.

The filter h(t) can be visualized as convolutional encoder [Gou-72] using its transversal filter representation shown in Fig. 4.2. The corresponding trellis for constraint length L = 3 is shown in Fig. 4.3. The trellis assumes a fixed periodic structure after a depth L. After this point, each of the 4 nodes (in general 2^{L-1} nodes) at time k are reached from two nodes corresponding to time (k-1). Thus there are two paths corresponding to two data sequences entering a given node, j ; j = 1,2,3, ... 2^{L-1}. The Viterbi algorithm calculates the probabilities of transition to the node j, the incremental log likelihood functions $\delta \Lambda_{k,j}$, in the interval (k+1) from various nodes in the interval k. These $\delta\Lambda_{k,j}$ are used in updating the log likelihood functions $\Lambda^{i}_{k,j}$ of the path i ; i = 1,2, entering the node j. The algorithm eliminates all but the most likely path from further considerations by retaining the path with largest $\Lambda^{i}_{k,j}$. Thus at time (k+1) there are 4 (in general 2^{L-1}) retained paths or survivors with log likelihood functions

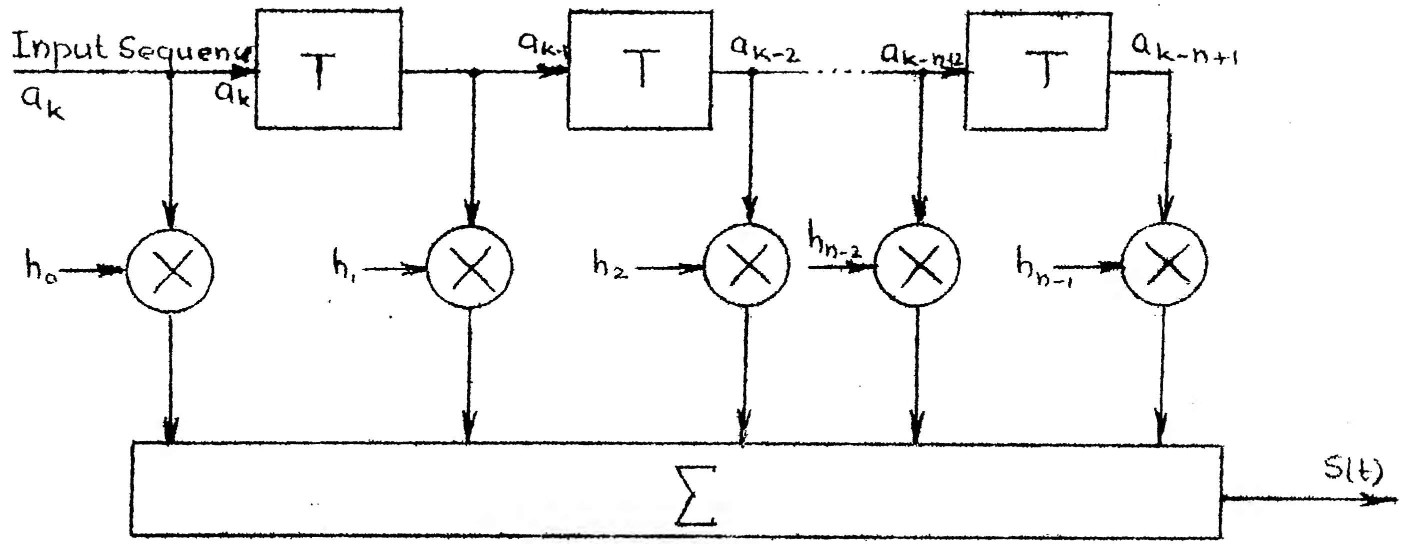


Fig. 4.2 Tapped delay line model of the channel.

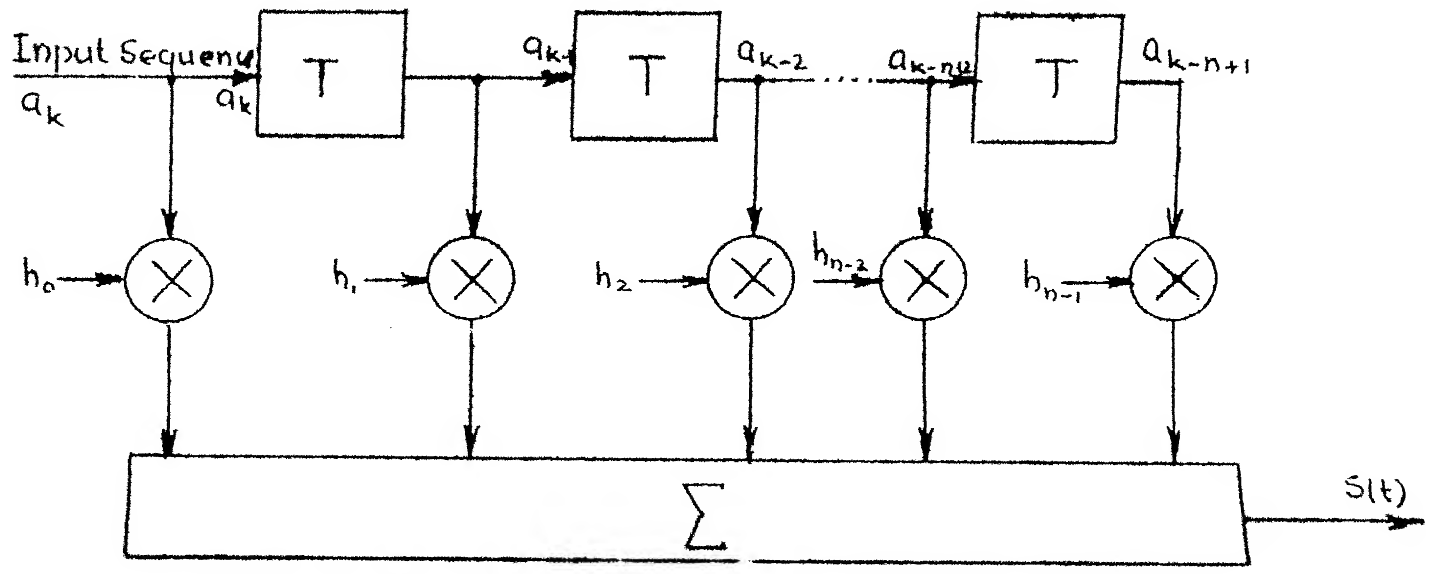
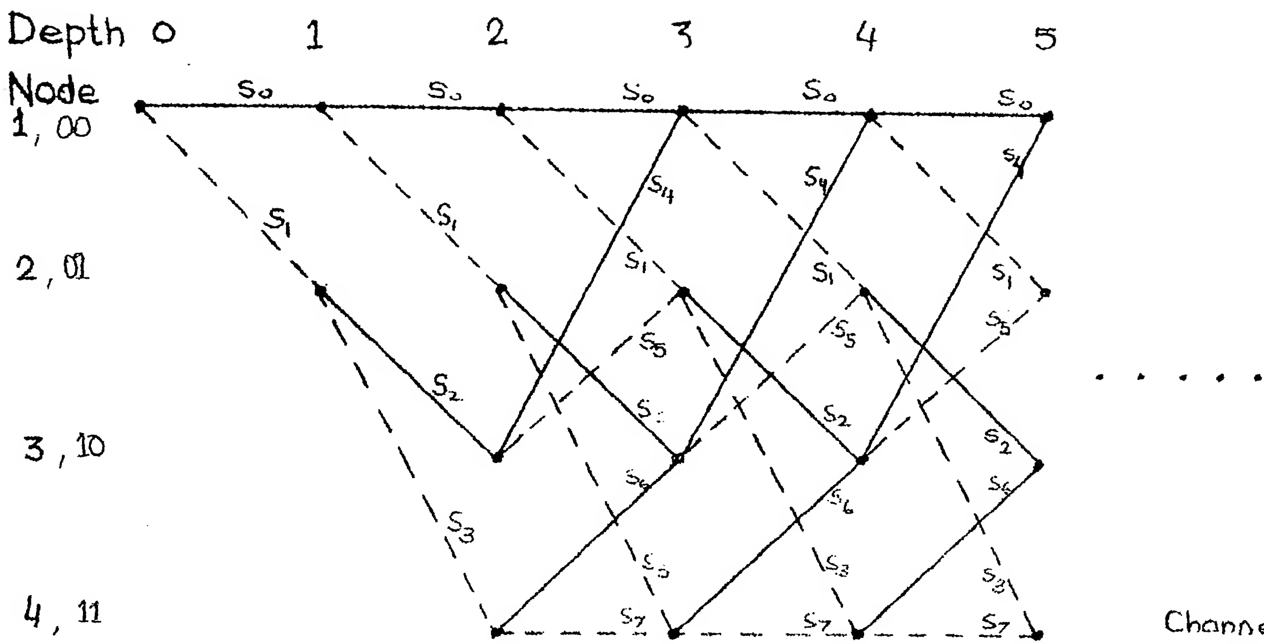


Fig. 4'3. Trellis diagram.



Channel Output

$$s(a_{k-2}, a_{k-1} a_k) = \sum_{i=0}^{2} a_{k-i} h_i$$

$S_0 = S(000)$ $\quad S_4 = S(100)$

$S_1 = S(001)$ $\quad S_5 = S(101)$

$S_2 = S(010)$ $\quad S_6 = S(110)$

$S_3 = S(011)$ $\quad S_7 = S(111)$

Fig. 4.3. Trellis diagram.

Viterbi [Vit-71] has shown that a decision delay of greater than 5.8 times the constraint length is sufficient and results in only negligible degradation of the performance.

4.3 Performance Analysis

4.3.1 Probability of Error Performance

The Viterbi decoder is a non-linear decoder and its performance can not be analyzed in closed form. Certain bounds on its performance have been obtained by Viterbi [Vit-71] and Forney [For-72]. Forney has shown that at medium and high SNR's the symbol error probability is over-bounded by

$$P(\varepsilon) \leq K_u Q[d_{min}/2\sigma]$$

where K_u is a small constant, d_{min} is the minimum energy of any non-zero signal and σ^2 is the spectral density of the noise and $Q(\cdot)$ is the probability of error function.

On the other hand, for any estimator the symbol error prob. is lower bounded by

$$P(\varepsilon) \geq K_L Q[d_{min}/2\sigma]$$

where K_L is another constant, typically within an order of magnitude of K_u. Thus the probability of error is tightly bounded by

$$K_L Q[d_{min}/2\sigma] \leq P(\varepsilon) \leq K_U Q[d_{min}/2\sigma]$$

As K_U and K_L are quite close, this result implies that with Viterbi decoding on most channels ISI does not lead to any significant degradation in performance, and the performance is almost optimum. In fact, the structure is optimum for ML estimation of the entire transmitted sequence [For-72].

4.3.2 Implementation of the Viterbi Decoder

The decoder may be constructed in a completely parallel or serial configuration or in a mixed configuration. In the parallel processing 2^L magnitude devices (adder + storage) and 2^{L-1} comparators are required. The fastest computation speeds are $1/T$ Hz. In serial configuration only one magnitude device and one comparator are needed but the fastest computation speeds are $2^L/T$ Hz. In serial/parallel configuration speeds of $2^L/xT$ are required where x is a complexity factor which depend on the proportion of serial to parallel circuitry.

For data rates in the range of Mbits/sec. the hardware requirements of a Viterbi decoder are unrealistic with present day technology in respect of speeds of operations involved. The data rates involved in transmission over telephone channels suggest possible use of Viterbi decoding on such channels. But on these voiceband channels limitation arises due to large ISI. For these channels L may

be as large as 40 or 50. Though high speeds of operation are not required a large number of magnitude devices and comparators are needed and the system is not economical.

4.4 VA Receiver Using Circulating Filters

As mentioned earlier the input to the Viterbi decoder is obtained by sampling the output of a whitened matched filter w(t) whose input is the received signal r(t).

$$z_k = \int_{-\infty}^{\infty} r(t)\, w(t-kT)\, dt.$$

The whitened matched filter is shown to be a cascade of a filter matched h(-t) to the channel h(t) and a transversal filter, whose tap gains are given by the sequence $1/f(D^{-1})$ where $f(D^{-1})$ is defined as

$$f(D^{-1}) \doteq f_0 + f_1 D^{-1} + f_2 D^2 + \dots + f_{n-1} D^{-(n-1)}$$

It is pointed out in [For-72] that the transversal filter is realizable when $f(D^{-1})$ has no roots on or inside unit circle. Knowledge about channel impulse response is needed in both the matched filter and the transversal filter.

In Fig. 4.4 a VA receiver scheme which uses circulating filter in estimating the channel impulse response is shown. The channel estimates obtained by the circulating filters are used in whitened matched filter in setting the tap gains of the transversal filter, and also in matched

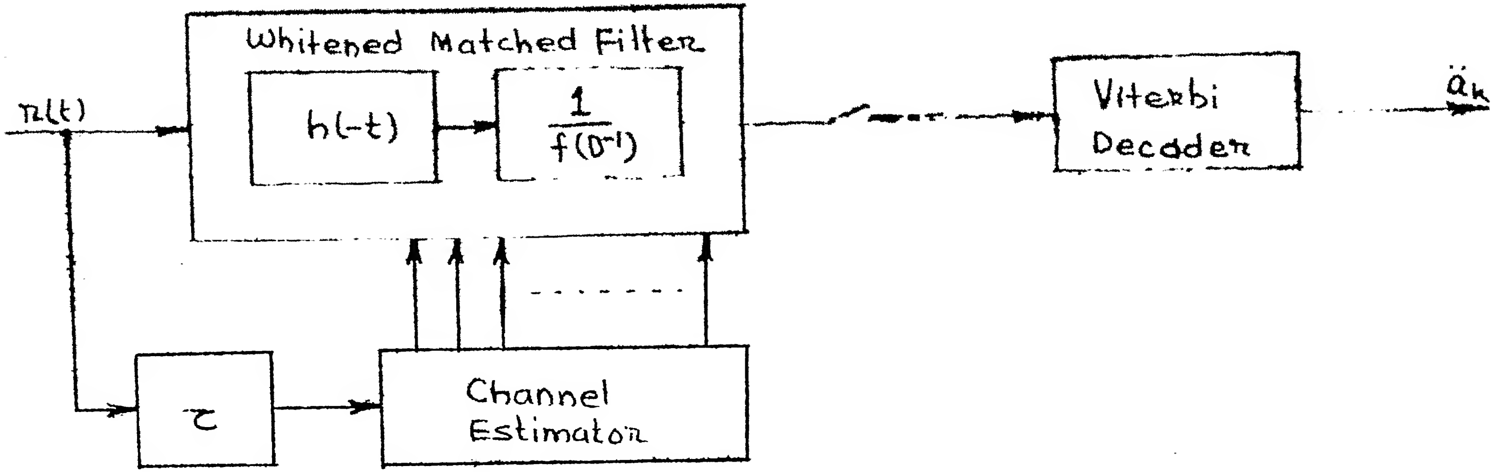


Fig 4.4. Adaptive VA Receiver.

filtering the received signal. As a decision delay τ of 6 to 7 times the constraint length is involved in the receiver decisions the channel estimates are available after a delay of τ sec. and the assumption to be made is that the characteristics do not vary significantly during this period. The assumption is valid for telephone channels as well as for high data rate transmission over troposcatter channels. The whitened matched filter output signal is then operated upon by the Viterbi decoder to obtain the ML estimates of the transmitted symbols.

For slowly fading channels it may not be required to adjust the tap gains of the transversal filter every T sec. (Adjusting tap-gains involves complicated complex calculations). The tap gains can be adjusted every pT bits where p may be a few hundred depending on data rate and fading characteristics of the channel. This reduces the speed requirements on hardware needed. In simulation it reduces computation time.

4.5 Simulation Results

Details of simulation strategy followed are given in the Appendix 1. Due to large amount of computation time involved only one 'run' of the program could be obtained in

which no errors were encountered for 6×10^5 data bits. The SNR was 6 dB and diversity reception was not used. The channel was an unknown random dispersive channel.

Some results were also obtained for telephone channel applications. These results along with the samples of channel response used are given in Table 5.

Table 5

Performance of Viterbi decoding in telephone channel equalization

SNR(dB)	No. of errors in 10^4 bits*
5	26
10	2
15	1

* Results averaged over 5 different noise sequences for same input data string

Telephone channel characteristic -

40.1, -8.3, -16.6, 3.0, 19.6, -0.5

CHAPTER 5

CONCLUSIONS

In this thesis we developed an adaptive decision-directed channel estimation technique using circulating filters and studied its performance in two different equalization schemes for troposcatter channels. The equalization schemes selected, the decision feedback equalization and the Viterbi decoding, are representative examples of two different classes of nonlinear equalization techniques. The first technique is the best among the techniques practical with present day technology and the second one is an optimum technique which is implementable too.

The channel estimators using circulating filters have the desirable property of suppressing noise. The theoretical derivation indicates that the circulating filters can suppress the noise in channel estimates by more than 12 dB. This substantial noise suppression is verified by simulation results. The capabilities of the scheme are further manifested by the simulation results that small time jitters and error propagation due to small bursts of errors (upto 5 errors) have no apparant effects on channel estimates.

A DFE receiver scheme for troposcatter channels which has no forward filter is developed by taking advantage of

highly skewed nature of the delay power spectrum of the channel. The receiver thus does not have any performance loss due to inadequately synthesized forward filter [c.f., Monsen's Structure, Mon-74]. Simulation results for such a DFE receiver using circulating filters for channel estimation, though inadequate for making any definite observations, suggest that the system has a quite low probability of error. The results without diversity compare well with the theoretical performance bounds. Simulated error propagation effects are found to cause negligible performance degradation for small bursts of errors. Furthermore, small time jitters of $\pm$ T/4 are seen to have insignificant effect on error rate.

The simulation results for Viterbi decoding using circulating filters are quite insufficient and no concrete conclusions about the receiver performance for troposcatter channels can be derived except that, possibly, the decoder is capable of performing well in an unknown channel condition.

Unfortunately, due to large amount of computation times involved sufficient data for making conclusive observations about the system performance could not be obtained. Certain assumptions had to be made to reduce the computation times required. Thus, in simulation of DFE receiver error propagation effects were ignored in the computation of the average probability of error. That this is a valid assumption for low

error-rate systems is justified by simulation results on error propagation effects. In Viterbi decoding it was assumed that the adaption of the whitened matched filter once in a thousand bits is sufficient. For high data rates of transmission over slowly fading channels like troposcatter channels this is a reasonable assumption.

The simulation programs not requiring much computation time were run on IBM-7044 computer at I.I.T, Kanpur. The simulation results on average probability of error were obtained on DEC-10 system at TIFR, Bombay. It took 9 hours of computation time on DEC-10 system to obtain results for DFE receiver. Another one hour was taken by VA receiver program. Total cost incurred in simulation on DEC-10 system was six thousand rupees.

One clear possibility of further work on the problem considered in this thesis is to obtain simulation data for more channel realizations and larger data sequences and to study the adaptive behaviour of the receivers by varying the delay power spectrum profile. Since time synchronization is extremely difficult in tropochannel applications it is desirable to study in detail the effects of time-jitter on the performance of the equalizers.

On the theoretical side work can be done on developing a DFE scheme which utilizes all the received signal energy without suffering any performance degradation due to inefficient forward filtering.

REFERENCES

[AF-70] Abend, K., and Fritchman, B.D., 'Statistical detection for communication channels with intersymbol interference', Proc. IEEE 58, May 1970, 779-785.

[Aus-67] Austin, M.E., 'Decision-feedback equalization for digital communication over dispersive channels', M.I.T. Lincoln Lab., Mass., Tech. Rep. 437, Aug.1967; also Sc.D. thesis, MIT, Cambridge, May 1967.

[BD-65] Bennet, W.R. and Davey, J.R., 'Data Transmission', McGraw-Hill, New York, 1965.

[Bel-63] Bello, P.A., 'Characterization of random time-variant linear channels', IEEE Trans. Commun.Systems. CS-11, December 1963.

[Bel-69] Bello, P.A., 'A troposcatter channel model', IEEE Trans. Commun. Tech. COM-17, April 1969, 130-137.

[Bra-70] Brady, D.M., 'An adaptive coherent diversity receiver for data transmission through dispersive media', IEEE Internat. Communications Conf. Rec. (San Francisco, Calif.) June 1970.

[BT-67] Berger, T., and Tufts, B.W., 'Optimum pulse amplitude modulation - Pt I : Transmitter - receiver design and bounds from information theory', IEEE Trans. Inform. Theory IT-13, April 1967, 196-208.

[CB-76] Cantoni, A., and Butler, P., 'Stability of decision feedback inverses', IEEE Trans. Commun. COM-24, September 1976, 970-977.

[Cha-50] Chalk, J.H.H., 'The optimum pulse shape for pulse communication', Proc. IEE 97, March 1950, 88-92.

[DS-65] DiToro, D.M., and Steiglitz, K., 'Application of the maximum principle to the design of minimum bandwidth pulses', IEEE Trans. Commun. Tech. COM-13, Dec. 1965, 433-438.

[Eri-71] Ericson, T., 'Structure of optimum receiving filters in data transmission systems', IEEE Trans. Inform. Theory IT-17, May 1971, 352-353.

[Eri-73] Ericson, T., 'Optimum PAM filters are always band limited', IEEE Trans. Inform. Theory IT-19 July 1973, 570-573.

[For-72] Forney, G.D., Jr., 'Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference', IEEE Trans. Inform. Theory IT-18, 5, May 1972, 363-378.

[For-73] Forney, G.D., Jr., 'The Viterbi algorithm', Proc. IEEE 61, March 1973, 268-278.

[Gab-46] Gabor, D., 'Theory of communication I,' The analysis of information', JIEE 93, Pt III, November 1946, 429-441.

[Ger-69] Gersho, A., 'Adaptive equalization of highly dispersive channels for data transmission', Bell System Tech. J. 48 January 1969 , 55-70.

[GBS-71] George, D.A., Bowen, R.R., and Storey, J.R., 'An adaptive decision feedback equalizer', IEEE Trans. Commun. Tech. COM-19, June 1971 , 281-293.

[GD-61] Gerst, I. and Diamond, J., 'The elimination of interference by input signal shaping', Proc. IRE, 49 July 1961, 1195-1203.

[Gou-72] Goutman, M.M., 'Intersymbol interference as a natural code', IEEE Trans. Commun. COM-20, October 1972 , 1033-1038.

[Han-71] Hansler, E., 'Some properties of transmission systems with minimum mean-square error', IEEE Trans. Commun. COM-19, August 1971 , 576-579.

[Hel-60] Helstrom, C.A.W., 'Statistical Theory of Signal Detection', Pergamon, New York, 1960, Sect. IV.5.

[Kai-60] Kailath, T., 'Correlation detection of signals perturbed by a random channel', IRE Trans. Inform. Theory IT-6, June 1960, 361-366.

[Kal-40] Kallman, H.E., 'Transversal filters', Proc. IRE-28, July 1940, 302-310.

[Kob-71A] Kobayashi, H., 'Application of probablistic decoding to digital magnetic recording systems', IBM J. Res. Develop. 15, Jan 1971, 64-74.

[Kob-71B] Kobayashi, H., 'Correlative level coding and maximum likelihood decoding', IEEE Trans. Inform. Theory IT-17, September 1971, 585-594.

[LSW-68] Lucky, R.W., Salz, J., and Weldon, E.J., 'Principles of Data Communication', McGraw-Hill, New York, 1968.

[Luc-65] Lucky, R.W., 'Automatic equalization for digital communication', Bell System Tech. J. 44, April 1965, 547-588.

[Luc-66] Lucky, R.W., 'Techniques for adaptive equalization of digital communication systems', Bell System Tech. J. 44, February 1966, 255-286.

[MF-76A] Magee, F.R. and Falconer, D.D., 'Evaluation of decision feedback equalization and Viterbi algorithm detection for voiceband data transmission: Part I', IEEE Trans. Commun. COM-24, October 1976, 1130-1139.

[MF-76B] Magee, F.R. and Falconer, D.D., 'Evaluation of decision feedback equalization and Viterbi algorithm detection for voiceband data transmission: Part II', IEEE Trans. Commun. COM-24, November 1976, 1238-1245.

[MP-73] Magee, F.R., and Proakis, J.G., 'Adaptive maximum likelihood sequence estimation for digital signaling in the presence of intersymbol interference', IEEE Trans. Inform. Theory IT-19, January 1973, 120-124.

[Mon-71] Monsen, P., 'Feedback equalization for fading dispersive channels', IEEE Trans. Inform. Theory IT-17, January 1971, 56-64.

[Mon-73] Monsen, P., 'Digital transmission performance on fading dispersive diversity channels', IEEE Trans. Commun. COM-21, January 1973, 33-39.

[Mon-74] Monsen, P., 'Adaptive equalization of the slow fading channel', IEEE Trans. Commun. COM-22, August 1974, 1064-1075.

[Mon-70] Morrissey, T.N., 'Analysis of decoders for convolutional codes by stochastic sequential machine method', IEEE Trans. Inform. Theory IT-16, July 1970, 460-469.

[MR-73] Monsen, P. and Richman, S.H., 'Adaptive data transmission study', Rome Air Develop. Center, Rome, N.Y. RADC Final Rep. TR-73-268, AD-768999, August 1973.

[Nyq-28] Nyquist, H., 'Certain Topics in Telegraph Transmission Theory', Trans. AIEE 47, February 1928, 617-644.

[Omu-69] Omura, J.K., 'On the Viterbi decoding algorithm', IEEE Trans. Inform. Theory IT-15, January 1969, 177-179.

[Pan-72] Panter, P.F., 'Communication system design : Line-of-sight and Troposcatter Systems, McGraw Hill, New York, 1972.

[PM-69] Proakis, J.G. and Miller, J.H., 'An adaptive receiver for digital signaling through channels with intersymbol interference', IEEE Trans. Inform. Theory, IT-15, July 1969, 484-497.

[QN-73] Qureshi, S.U.H. and Newhall, E.E., 'An adaptive receiver for data transmission over time-dispersive channels', IEEE Trans. on Inform. Theory, IT-19, July 1973, 448-457.

[SBS-66] Schwartz, M., Bennet, W.R., and Stein, S. Communication Systems and Techniques, McGraw-Hill, New York, 1966.

[Smi-65] Smith, J.W., 'The joint optimization of transmitted signal and receiving filter for data transmission systems', Bell System Tech. J. 44, December 1965, 2363-2392.

[TS-64] Tufts, D.W. and Shnidman, D.A., 'Optimum waveforms subject to both energy and peak value constraints' Proc. IEEE 52, September 1964, 1002-1007.

[Tuf-65] Tufts, D.W., 'Nyquist's problem - The joint optimization of transmitter and receiver in pulse amplitude modulation : Proc. IEEE 53, March 1965, 248-259.

[Van-68] VanTrees, H.L., 'Detection, Estimation, and Modulation Theory, Part I, Wiley, New York, 1968.

[Vit-67] Viterbi, A.J., 'Error bounds for convolutional codes and an asymptotically optimum decoding algorithm', IEEE Trans. Inform. Theory IT-13, April 1967, 260-267.

[Vit-71] Viterbi, A.J., 'Convolutional codes and their performance in communication systems', IEEE Trans. Commun. Technol. COM-19, October 1971, 751-772.

[Wid-66] Widrow, B., 'Adaptive filters, I : Fundamentals', Stanford, Electronics Lab., Stanford, Univ., Stanford, Calif., Tech. Rept. 6764-6, December 1966.

[WZ-65] Wozencraft, J.M. and Jacobs, I.M. Principles of Communication Engineering, Wiley, New York, 1965.

APPENDIX 1

PERFORMANCE SIMULATION

The objective of the computer simulation was to study the behaviour of an adaptive channel estimator using circulating filters and to evaluate for troposcatter channels the performance of a decision feedback equalizer and a Viterbi decoder with circulating filters for channel estimation. Sampled versions of continuous-time systems were considered. A trade-off between number of samples per baud and computation time required was found necessary. The simulation was carried out with 4 (M) samples/baud.

As described in Chapter 2, a troposcatter channel is completely described by specification of its delay power spectrum. The channel impulse response is selected from an ensemble defined by an approximation to the delay power spectrum; $Q(\tau) = \alpha^2 \tau e^{-\alpha\tau}$. The complex zero-mean uncorrelated Gaussian samples with variance equal to the delay power spectrum at the sampling instants constitute the channel impulse response. The channel impulse response is finite with a length of 3(N) bauds.

The data input to the channel is a pseudo-random binary sequence with symbol separation T normalized to unity. The sequence is obtained by generating a random number using the

subprogram RNDYI (RAN on DEC-10 system). If the random number is greater than 0.5 the input symbol is taken as +1, otherwise -1.

The channel output is the convolution of the transmitted data sequence and the channel pulse response. The channel pulse response is obtained by convolving channel impulse response with a unit pulse of normalized baud duration. The received signal is then obtained by adding a zeromean white Gaussian noise sample with variance σ^2 to the channel output. The Gaussian noise sample n_k is generated by using the Box-Muller formula

$$n_k = \sigma[-2\ln (R_1)]^{\frac{1}{2}} \cos(2\pi R_2)$$

where R_1 and R_2 are uncorrelated uniformly distributed random numbers in the range 0 and 1. The noise variance is adjusted to fix the SNR of the received signal.

The steps involved in obtaining the received signal are given in flow-chart 1.

In the following section, the estimator behaviour in extracting the channel information from the received signal is studied. In the subsequent sections the decision feedback equalizer and the Viterbi decoding schemes with circulating filters are simulated and their performance evaluated.

A 1.1 Channel Estimation Scheme

The estimator is used to determine the channel pulse response. For a channel impulse response of duration N(=3), two sets of (N+1) circulating filters; one for the quadrature component and the other for in phase component of the received signal, are needed.

The simulation steps are shown in Flow chart 2.

The sample at time T/4 of a given chip, $h_j^c(t)$ is estimated in the jth circulating filter in the first iteration and the sample at time T/2 in the second iteration and so on. In estimating the samples of the chip $h_j^c(t)$ the interference contribution due to other chips is subtracted from the received signal sample using past decisions and channel estimates in the preceding iteration. Correlation with the symbol $\ddot{b}_{k-j}$ gives a noisy channel estimate. The circulating filter averages these estimates in successive iterations by adding its input signal to the filter output signal in the preceding iteration weighted by feedback gain f_j. The channel estimates are given by circulating filter output weighted by $(1-f_j)$.

The noise suppression due to the circulating filter was studied by computing the difference in SNR's at the filter output and input. Effects of decision error on channel estimates were investigated by deliberately

introducing decision errors. The time-jitter effects were evaluated by shifting the received signal in steps of $\pm$ T/4.

A 1.2 Decision Feedback Equalization

The simulation steps are shown in Flow chart 3. The samples of the received signal are generated as described earlier. The channel estimates are obtained from the received signal using receiver decisions. The channel response estimates and past decisions of the receiver are used in computing the past symbol interference. The signal samples are then multiplied by the corresponding samples of the channel response in the first T sec. The resulting signals are summed up to form sampled output of the matched filter. If it is greater than 0 the decoded bit is + 1 otherwise -1. The receiver decision is checked against the transmitted bit to detect decision errors.

The steps are repeated over a large number of input data bits and also for different realizations till sufficient data for computation of the probability of error is obtained.

The performance was studied mainly in the context of evaluating the average probability of error and studying the effects of error propagation and time jitter. The last two effects were studied by deliberately introducing decision errors and time-jitters in the received signal respectively

The computation of error probability required a very large amount of the computation time. To reduce this time certain assumptions were made. Thus, it was assumed that the receiver decisions are essentially correct, and the error propagation was not considered. Another assumption was that in diversity reception there is a very low probability of decision error when a correct decision is obtained in one branch. This implies that the need for considering all diversity branches arises only when an error occurs on a single branch.

A 1.3 Viterbi Decoding

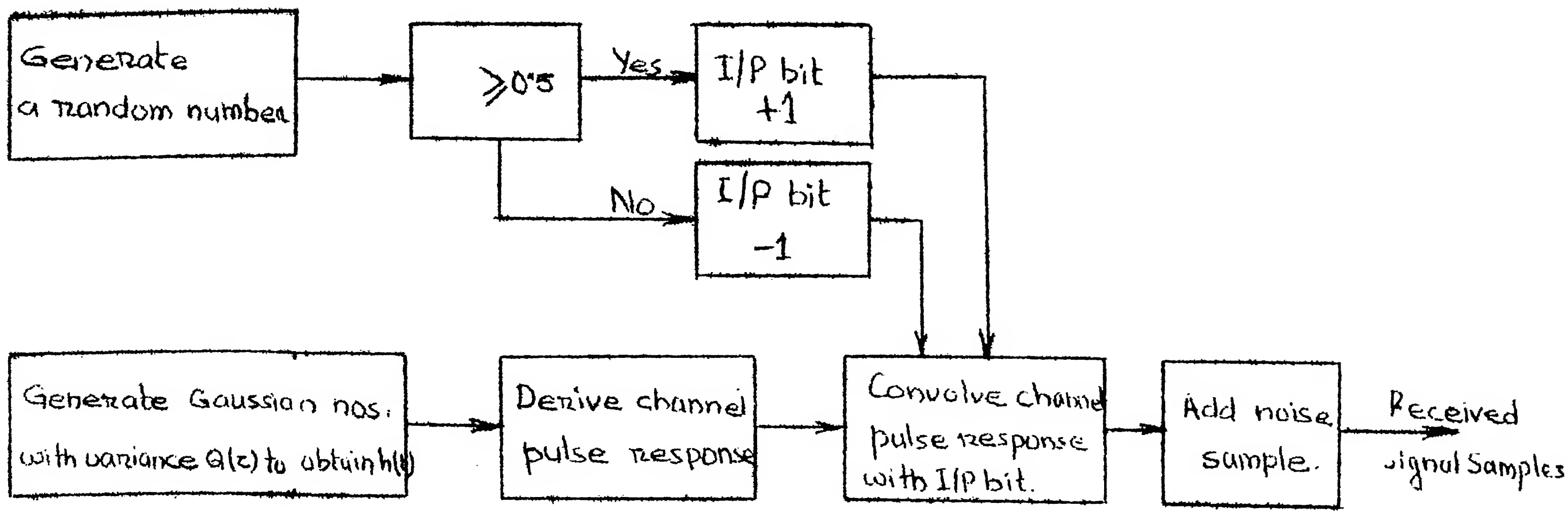
The flow diagram of simulation is shown in Flow chart 4. The channel impulse response estimates are used in the computation of the whitening sequence f(D). First, the sequence R(D), autocorrelation function of impulse response h(t), is obtained by convolution of h(t) with itself. The roots of the polynomial equation R(D) are then computed and the sequence f(D) is formed by collecting those roots which lie outside the unit circle; i.e. which have magnitude greater than unity.

Since computation of the sequence f(D) involves complicated complex calculations, the sequence is updated only once in a thousand bits. It is assumed that the data transmission is at sufficiently high speed over a slowly fading channel, so that the channel characteristics do not

change significantly in the thousand bits duration.

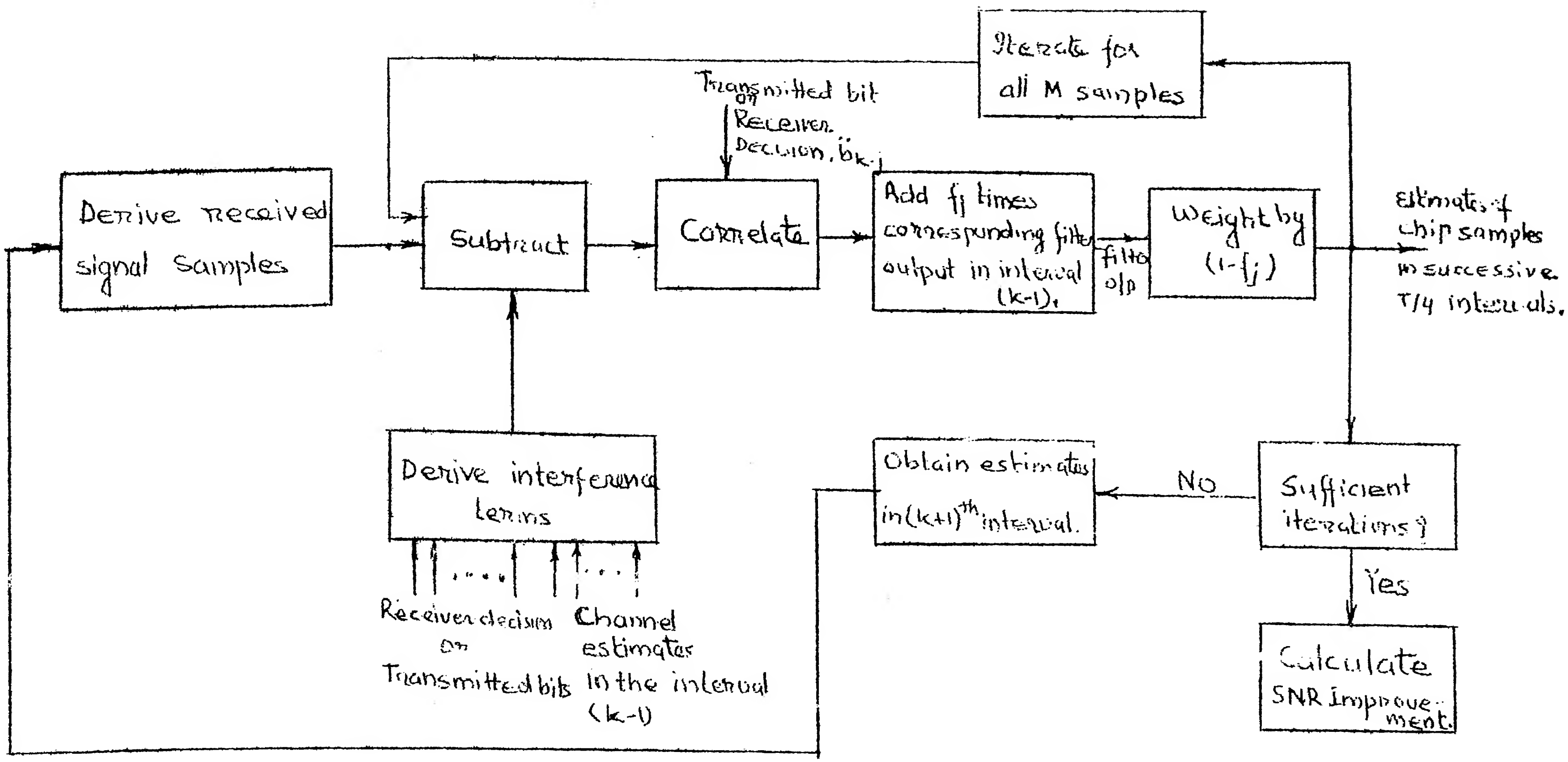
The input to the Viterbi decoder is then obtained by convolving the input data sequence with the sequence f(D) and adding a white Gaussian noise sample to it.

As described in Chapter 4, the Viterbi algorithm proceeds recursively in two steps. It calculates the log likelihood functions for all allowable paths leading to a given node in kth iteration by calculating the associated probability of transition and adding it to the log likelihood of the path in $(k-1)^{th}$ interval. In the second step it chooses the path with maximum log likelihood as the survivor. As derived in [Vit-71], for BPSK transmission, the probability of transition is given by the product of the code symbol and received sequence. The path histories, which are in the form of sequences of +1 and -1 are stored in their decimal equivalent form. The decoding delay is kept at 7 times the channel impulse response spread and the bit corresponding to the first node is taken as the receiver delay.

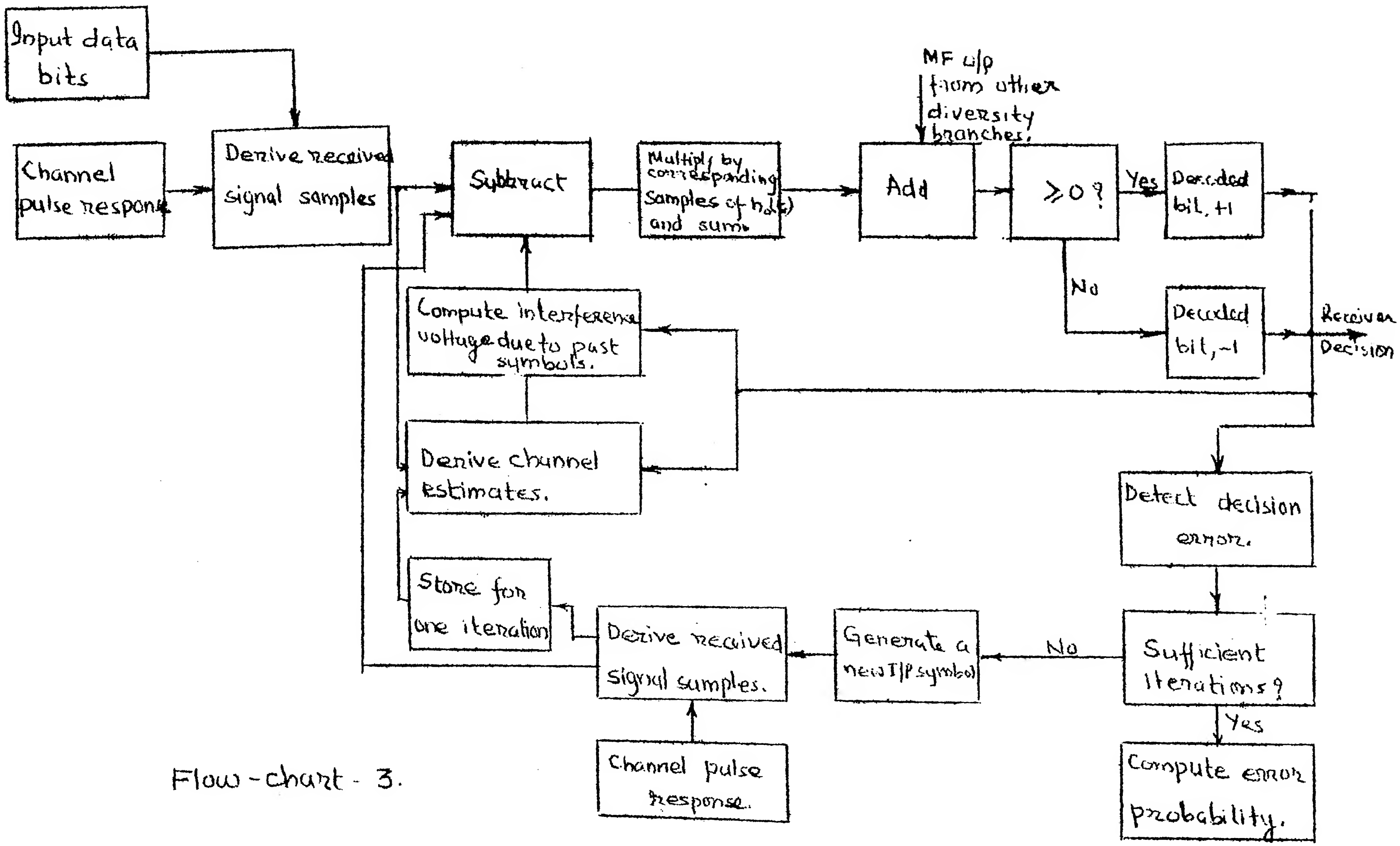


Generate a random number
≥0·5
Yes
No
I/P bit +1
I/P bit −1
Generate Gaussian nos. with variance Q(z) to obtain h(t)
Derive channel pulse response
Convolve channel pulse response with I/P bit.
Add noise sample.
Received signal Samples

Flow Chart 1.

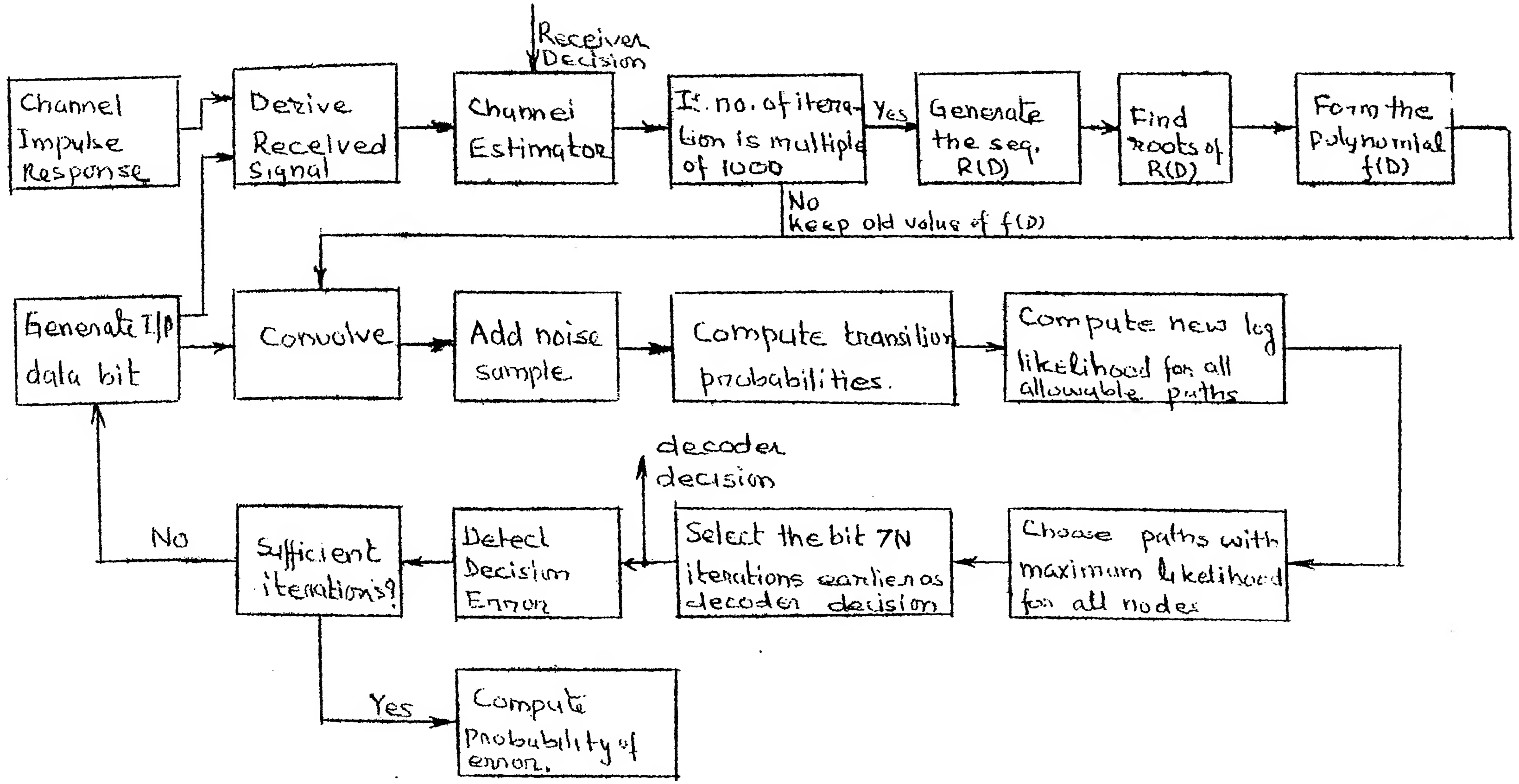

Derive received signal samples
Subtract
Correlate
Transmitted bit or Receiver Decision, $\hat{b}_{k-1}$
Add f_j times corresponding filter output in interval (k-1).
filter o/p
Weight by $(1-f_j)$
Estimates of chip samples in successive T/4 intervals.
Iterate for all M samples
Derive interference terms
Receiver decision on Transmitted bits
Channel estimates in the interval (k-1)
Obtain estimates in $(k+1)^{th}$ interval.
No
Sufficient iterations?
Yes
Calculate SNR Improvement.

Flow - Chart - 2.



Input data bits
Channel pulse response
Derive received signal samples
Subtract
Multiply by corresponding samples of h(t) and sum
MF o/p from other diversity branches.
Add
≥0 ?
Yes
Decided bit, +1
No
Decided bit, -1
Receiver Decision
Compute interference voltage due to past symbols.
Derive channel estimates.
Store for one iteration
Derive received signal samples.
Channel pulse response.
Generate a new I/P symbol
No
Detect decision error.
Sufficient Iterations?
Yes
Compute error probability.

Flow-chart - 3.



Channel Impulse Response
Derive Received Signal
Receiver Decision
Channel Estimator
Is no. of iteration is multiple of 1000
Yes
Generate the seq. R(D)
Find roots of R(D)
Form the Polynomial f(D)
No
Keep old value of f(D)
Generate I/P data bit
Convolve
Add noise sample
Compute transition probabilities.
Compute new log likelihood for all allowable paths
decoder decision
No
Sufficient iterations?
Detect Decision Error
Select the bit 7N iterations earlier as decoder decision
Choose paths with maximum likelihood for all nodes
Yes
Compute Probability of error.

Flow Chart – 4

```
MULATION OF A DIGITAL COMMUNICATION SYSTEM FOR COMMUNI
TION OVER A FADING DISPERSIVE(TROPO-SCATTER) CHANNEL.
VARIATION OF THE DECISION FEEDBACK EQUALIZATION
CHNIQUE WHICH TAKES ADVANTAGE OF THE ASSYMMETRIC
TURE OF THE TROPO-CHANNEL CHARACTERISTIC IS USED.
ECULATING FILTERS ARE USED FOR OBIAING CHANNEL ESTIMATES.

VERSITY RECEPTION OF THE ORDER OF FOUR IS USED.
ME-JITTER IS NEGLECTED.
POR PROPAGATION IS NEGLECTED.

ERAGE RATE OF ERROR VS AVERAGE SNR CHARACTERISTIC OF
E SYSTEM IS DETERMINED.
ERAGE ERROR PROBABILITY CHARACTERISTIC WITHOUT DIVERSITY
 ALSO OBTAINED.

E PROGRAM CAN BE RESTARTED IF SOME FAILURE OCCURS
 BETWEEN A RUN.
E PROGRAM IS SUITABLE ONLY WHEN CERTAIN INITIALIZATION
LUES ARE KNOWN.
E INITIALIZATION VALUES ARE:DATA SEQUENCE,A,LAST RANDOM
MBER IRAN,NO OF ERRORS IN PREVIOUS ITERATIONS IERR AND K3
D NOISE SAMPLES WNOISE AND OPNOISE.


  INTEGER CHANEL
  DIMENSION G(5),A(10),F(4,2),F1(4,2),S(40,4,2),
  OPNOIS(4,2),WNOIS(4,2),SS(10,4,2),SIGMA(4),SUM(4,10),SNR(4),
  IERR(4,10),K3(4,10),WSIGMA(4),WDSIGM(4),ONOIS(4)
  PI=8.*ATAN(1.)

: NO OF SAMPLES/BAUD. N= CH. IMPULSE RESPONSE SPREAD.
=NO OF DIVERSITY BRANCHES.  CHANEL=NO OF DIFFERENT
ANNEL REALIZATIONS.

  READ(5,10)M,N,L1,CHANEL,ALPHA
  FORMAT(4I5,F12.6)
```

```
      NN=M*M
      M1=M+1
      FINV=0.1
      NINA=10
      NINI=NINA-1
      NUM=NINA-M+1
      T3=1.0
      READ(5,11)IRAN
      FORMAT(I20)
      READ(5,12)(A(I),I=1,NINA)
      FORMAT(10F5.1)
      READ(5,13)((RNOIS(I,I1),OPNOIS(I,I1),I=1,M),I1=1,2)
      FORMAT(8F10.5)
      READ(5,14)((IERR(I,I1),K3(I,I1),I1=1,CHANEL),I=1,4)
      FORMAT(10I5)

MPLES OF THE DELAY POWER SPECTRUM ARE CALCULATED.

      DM=1./FLOAT(M)
      DO 20 I=1,M
      CI=I
      CP=CI*DM
      Q(I)=CP*EXP(-ALPHA*CP)*ALPHA*ALPHA

ANNEL IMPULSE RESPONSE.
NO OF DIFFERENT CHANNEL REALIZATIONS ARE OBTAINED.

      DO 40 I2=1,CHANEL
      DO 40 I3=1,L1
      II=(I2-1)*L1+I3
      DO 40 I1=1,2
      DO 40 I=1,M
      R3=RAN(DUM)
      R4=RAN(DUM)
      S(II,I,I1)=SQRT(Q(I))*SQRT(-2.0*ALOG(R3))*COS(PI*R4)
```

```
      READ(5,60)((F(I,I1),I=1,N1),I1=1,2)
 60   FORMAT(8F10.6)
     DO 170 II=1,CHANEL
      IJ=(II-1)*L1
      DO 170 I3=1,L1
      I2=IJ+I3
      DO 170 I1=1,2
      DO 170 I=2,M
170   S(I2,I,I1)=S(I2,I-1,I1)+S(I2,I,I1)
      T4=0.0
      H4=0.0

C  NOISE VARIANCES FOR DIFFERENT SNRS ARE CALCULATED.

      DO 240 I2=1,CHANEL
      IJ=(I2-1)*L1
      DO 240 I4=1,L1
      I3=IJ+I4
      DO 240 II=1,M
      DO 240 I1=1,2
240   H4=H4+S(I3,II,I1)*S(I3,II,I1)
      SIG=SQRT(H4/(2.0*FLOAT(M*L1*CHANEL)*10.**1.5))
      DO 260 I5=1,4
      SNR(I5)=15.0-T4
      SIGMA(I5)=SIG*T3
      T4=T4+3.0
360   T3=T3*SQRT(2.0)
      DO 300 I2=1,CHANEL
      I4=(I2-1)*L1+1
      DO 290 I3=1,M
      DO 290 I1=1,2
290   SS(I2,I3,I1)=S(I4,I3,I1)
300   CONTINUE
```

```
   THE SUBPROGRAM SETRAN SETS THE STARTING VALUE OF THE
   RANDOM NUMBER GENERATOR SUBPROGRAM RAN EQUAL TO ITS ARGUMENT

      CALI SETRAN(IRAN)

   THE SIMULATION PROGRAM IS EXECUTED OVER ONE MILLION
C  DATA SYMBOLS.

      DO 1000 KK1=1,100
      KJ=(KK1-1)*10000
      DO 1000 KK2=1,10000
      K=KJ+KK2
      DO 308 I=1,CHANLL
      DO 308 I1=1,4
308   SUM(I1,I)=0.0
      DO 310 J=2,NINA
310   A(J-1)=A(J)

   A NEW RANDOM DATA SYMBOL IS GENERATED.

      AX=RAN(DUM)
      IF(AX.GE.0.5) GO TO 320
      A(NINA)=-1.
      GO TO 330
320   A(NINA)=1.0
330   D=0.0
      DO 430 J2=MIN,NINA
430   D=D+A(J2)
      A2≡A(NINA)
     A3=A(NIN1)
```

```
NOISE CONTENT AT THE CIRCULATING FILTER O/P IS OBTAINED.
THE NOISE SAMPLES ARE GENERATED BY BOX MULLER FORMULA.

      DO 450 I1=1,2
      DO 450 II=1,L
      WNOIS(II,I1)=0.9*WNOIS(II,I1)+OPNOIS(II,I1)*A3
      WNOISE=WNOIS(II,I1)*FINV
      R1=RAN(DUM)
      R2=RAN(DUM)
      OPNO=SQRT(-2.*ALOG(R1))*COS(PI*R2)
      DO 420 I6=1,4
      ONOIS(I6)=OPNO*SIGMA(I6)
      WSIGMA(I6)=WNOISE*SIGMA(I6)
420   WDSIGM(I6)=D*WSIGMA(I6)
      DO 440 I3=1,CHANEL
      SS1=SS(I3,II,I1)
      SIGNAL=A2*SS1
      DO 440 I5=1,4

R IS CHANNEL O/P

      R=ONOIS(I5)+SIGNAL
      Y=SS1+WSIGMA(I5)

PAST SYMBOL INTERFERENCE IS REMOVED AND MATCHED
FILTERING IS DONE.

440   SUM(I5,I3)=SUM(I5,I3)+(R-WDSIGM(I5))*Y
450   OPNOIS(II,I1)=OPNO

AN ML ESTIMATE OF THE TRANSMITTED SYMBOL IS MADE AND
ERROR IN ESTIMATION ,IF ANY, IS DETERMINED.

      DO 480 I1=1,CHANEL
      DO 480 I=1,4
      A1=-1.0
```

```
      IF(SUM(I,I1).GE.0.0) A1=1.0
      IF(A2.EQ.A1) GO TO 480

    IN CASE OF A DETECTION ERROR DIVRSITY RECEPTION IS
    INCORPORATED IN THE SYSTEM AND THE DECISION ABOUT THE
    TRANSMITTED SYMBOL IS MADE AGAIN.

      K3(I,I1)=K3(I,I1)+1
      DO 520 I10=2,L1
      I11=(I1-1)*L1+I10
      DO 520 I12=1,2
      DO 520 I13=1,N
      WNISE=0.0
      DO 500 I14=1,NTPT
      R1=RAN(DUM)
      R2=RAN(DUM)
      OPNISE=SIGMA(I)*SQRT(-2.*ALOG(R1))*COS(PI*R2)
500   WNISE=0.9*WNISE+OPNISE*A(I14)
      WNISEE=WNISE*FIOV
      Y=S(I11,I13,I12)+WNISEE
      R1=RAN(DUM)
      R2=RAN(DUM)
      OPNISE=SIGMA(I)*SQRT(-2.*ALOG(R1))*COS(PI*R2)
      ROUT=OPNISE+A(NTNA)*S(I11,I13,I12)
520   SUM(I,I1)=SUM(I,I1)+(ROUT-D*WNISEE)*Y
      IF(SUM(I,I1).GE.0.0) GO TO 530
      A1=-1.0
      GO TO 540
530   A1=1.0
540   IF(A2.EQ.A1) GO TO 480
      IERR(I,I1)=IERR(I,I1)+1
480   CONTINUE
1000  CONTINUE
```

```
      WRITE(6,1500)K
 1500 FORMAT(/,10X,11HNO OF BITS=,I10)
      DO 1600 I=1,4
      WRITE(6,1510)SNR(I),(IERR(I,I1),K3(I,I1),I1=1,CHANEL)
 1510 FORMAT(10X,12HAVERAGE SNR=,F5.1,2HDB,/,20(1X,I5))
 1600 CONTINUE
      WRITE(6,1520)(A(I),I=1,NIMA)
 1520 FORMAT(10X,5HA(I)=,/,20(1X,F4.1))
      WRITE(6,1530)((WPOS(I,I1),OPDOS(I,I1),I=1,M),I1=1,2)
 1530 FORMAT(8(2X,F10.4))
      CALL SAVRAN(IRAN)
      WRITE(6,1550)IRAN
 1550 FORMAT(/,20H LAST RANDOM NO SEED,I20)
 9000 CONTINUE
      STOP
      END
```

```
      INTEGER OD1,EF,YY1,PATH(32)
      DIMENSION G(12),H(12,2),A(10),F(5,2),F1(5,2),B(5,2),
     1 S(5,5,2),X(5,5,2),X1(5,5,2),Y(5,5,2),T(5,5),H2(5,5),
     2 SNR(5,5),SNRC(5,5),H3(5,5,2),HH(12,2),SNRT(5,5),
     3 HAVE(5,5,2),FR(20),FI(20),E(10),SS(32),AS(32),IPATH(32),
     4 COF(50),ROOTR(20),ROOTI(20),AMID(20),ROOTFR(20),ROOTFI(20),V1(20)
     5 ,V2(20),VP(30),Y3(20),Y4(20),X3(20),X4(20),ZZ(20),ZZ1(20)
      PI=8.*ATAN(1.)
      READ(5,10)M,N,SIGMA,ALPHA
10    FORMAT(2I5,F12.6)
        WRITE(6,10)M,N,SIGMA,ALPHA
      M1=M-1
      NN2=2*N
      NN1=N-1
      L=2**N
      L1=L/2
      N3=7*N
      NN=M*N
      N1=N+1
      IERR=0
      FM=1./FLOAT(M)
      DO 20 I=1,NN
      CI=I
      CP=CI*FM
20    G(I)=CP*EXP(-CP*ALPHA)*ALPHA*ALPHA
        WRITE(6,30)(G(I),I=1,NN)
30    FORMAT(/,10X,28HDELAY POWER SPECTRUM SAMPLES,/,12(2X,F7.4))
      DO 40 I1=1,2
      DO 40 I=1,NN
      R3=RAN(DUM)
      R4=RAN(DUM)
40    H(I,I1)=SQRT(G(I))*SQRT(-2.*ALOG(R3))*COS(PI*R4)
       WRITE(6,50)((H(I,I1),I=1,NN),I1=1,2)
50    FORMAT(/,10X,32HCHANNEL IMPULSE RESPONSE SAMPLES,/,12(2X,F7.4))
       READ(5,60)((F(I,I1),I=1,N1),I1=1,2)
60    FORMAT(8F10.0)
        WRITE(6,70)((F(I,I1),I=1,N1),I1=1,2)
70   FORMAT(/,10X,50HTHE FEEDBACK PATH GAINS OF THE CIRCULATING FILTERS
    1,/,12(2X,F7.4))
```

```
      DO 90 I=1,M1
      R=RAN(DUM)
      IF(R.GE.0.5) GO TO 80
      A(I)=-1.0
      GO TO 90
80    A(I)=1.0
90    CONTINUE
        WRITE(6,100)(A(I),I=1,M1)
100   FORMAT(/,10X,21HFIRST M1 I/P SYMBOLS,/,6(10X,F4.1))
      DO 140 I1=1,2
      DO 140 I=1,M
      DO 140 I2=1,M1
      S(I,I2,I1)=0.0
      LM=I
      LINA=M
      IF(I2.EQ.1) GO TO 110
      IF(I2.NE.M1) GO TO 120
      IF(I.EQ.M) GO TO 140
      LINA=M-I
      GO TO 120
110   LINA=I
      LM=M
120   DO 130 J1=1,LINA
      LK=M*(I2-2)+LM+J1
      S(I,I2,I1)=S(I,I2,I1)+H(LK,J1)
130   CONTINUE
140   CONTINUE
        WRITE(6,150)(((S(I,I1,I2),I1=1,M1),I2=1,2),I=1,M)
150   FORMAT(4(5X,F8.3))
      H4=0.
      AM=2.*SIGMA*SIGMA
      AM1=0.
      DO 190 II=1,M
      DO 160 J1=1,2
      R1=RAN(DUM)
      R2=RAN(DUM)
160   B(II,J1)=SIGMA*SQRT(-2.*ALOG(R1))*COS(PI*R2)
      DO 190 I2=1,M1
      IF(I2.NE.M1) GO TO 170
      IF(II.EQ.M) GO TO 190
170   H1=0.
```

```
      J=N1+1-I2
      DO 18Ø I1=1,2
      B(II,I1)=B(II,I1)+A(J)*S(II,I2,I1)
 18Ø  H1=H1+S(II,I2,I1)*S(II,I2,I1)
      H4=H4+H1
      AM1=AM1+AM
      SNRI(II,I2)=1Ø.*ALOG1Ø(H1/AM)
 19Ø  CONTINUE
      SNRAVE=1Ø.*ALOG1Ø(H4/AM1)
        WRITE(6,2ØØ)((SNRI(I,I1),I1=1,N1),I=1,M)
 2ØØ  FORMAT(/,1ØX,3ØHI/P SNR OF CIRCULATING FILTERS,/,4(1ØX,F9.2,2HDB)
     1)
        WRITE(6,21Ø)SNRAVE
 21Ø  FORMAT(/,1ØX,16HI/P SNR(AVERAGE),/,F1Ø.4)
      DO 22Ø I=1,N1
      DO 22Ø I1=1,2
      F1(I,I1)=1.Ø+F(I,I1)+F(I,I1)*F(I,I1)
      DO 22Ø I2=1,M
      T(I2,I)=Ø.Ø
      H3(I2,I,I1)=Ø.Ø
      Y(I2,I,I1)=Ø.Ø
 22Ø  X(I2,I,I1)=Ø.Ø
      DO 225 I=1,L
      PATH(I)=Ø.
 225  SS(I)=Ø.
      DO 1ØØØØ KK1=1,1ØØ
      DO 1ØØØØ KK2=1,1ØØØØ
      K=(KK1-1)*1ØØØØ+KK2
      DO 27Ø I1=1,2
      DO 25Ø II=1,M
      C=Ø.Ø
      DO 23Ø I=1,N1
      J=N1+1-I
 23Ø  C=C+A(J)*Y(II,I,I1)
      DO 25Ø I=1,N1
      IF(I.NE.N1) GO TO 24Ø
      IF(II.EQ.M) GO TO 25Ø
 24Ø  J=N1+1-I
      X1(II,I,I1)=X(II,I,I1)*F(I,I1)+E(II,I1)*A(J)+B(J)*(A(J)*Y(II,I,I1
     1)-C)
 25Ø  CONTINUE
```

```
      DO 260 I=1,N1
260   F1(I,I1)=F(I,I1)*F1(I,I1)+1.
      DO 270 II=1,M
      DO 270 I=1,N1
      X(II,I,I1)=X1(II,I,I1)
270   Y(II,I,I1)=X(II,I,I1)/F1(I,I1)
      DO 280 I=1,M
      DO 280 I2=1,N1
      DO 280 I1=1,2
      H3(I,I2,I1)=H3(I,I2,I1)+Y(I,I2,I1)
280   T(I,I2)=T(I,I2)+(Y(I,I2,I1)-S(I,I2,I1))**2
290   KK=K/10000*10000
300   IF(K.NE.KK) GO TO 350
      AK1=K-
      DO 320 II=1,M
      DO 320 I=1,N1
      IF(I.NE.N1) GO TO 310
      IF(II.EQ.M) GO TO 320
310   H2(II,I)=H3(II,I,1)**2+H3(II,I,2)**2
      SNRO(II,I)=10.*ALOG(H2(II,I)/(T(II,I)*AK1))/2.3
      SNR(II,I)=SNRO(II,I)-SNRI(II,I)
320   CONTINUE
        WRITE(6,330)K,IERR
330   FORMAT(/,10X,2HK=,I6,4X,6HERROR=,I6)
        WRITE(6,340)((SNR(I,I1),I1=1,N1),I=1,M)
340   FORMAT(/,10X,15HSNR IMPROVEMENT,/,4(10X,F10.5,2HDB))
350   DO 360 I=2,N
360   A(I-1)=A(I)
      KK=K/1000*1000
      IF(KK.NE.K) GO TO 825
      KK1=K-N
      DO 370 I1=1,2
      DO 370 II=1,M
      DO 370 I=1,N1
370   HAVE(II,I,I1)=H3(II,I,I1)/FLOAT(KK1)
      DO 400 I1=1,2
      HH(1,I1)=HAVE(1,1,I1)
```

```
      DO 380 I2=2,M
380   HH(I2,I1)=HAVE(I2,1,I1)-HAVE(I2-1,1,I1)
      DO 400 I=2,M
      DO 400 I1=1,M
      J1=(I-1)*M+I1
      P=0.0
      DO 390 I2=1,M1
      J=(I-2)*M+I2+1
390   P=P+HH(J,I1)
400   HH(J1,I1)=HAVE(I,1,I1)-P
      DO 420 I=1,M
      I1=M+1-I
      I2=M+I-1
      RD(I1)=0.
      RD(I2)=0.
      DO 410 I3=1,I1
      DO 410 I4=1,M
      I5=(I3-1)*M+I4
      I6=(I3-2+I)*M+I4
      DO 410 I7=1,M
410   RD(I1)=RD(I1)+HH(I5,I7)*HH(I6,I7)
420   RD(I2)=RD(I1)
      CALL POLRT(RD,COF,MM2,ROOTR,ROOTI,IER)
      J=0
      DO 440 I=1,MM2
      AMID(I)=SQRT(ROOTR(I)*ROOTR(I)+ROOTI(I)*ROOTI(I))
      IF(AMID(I).GT.1.0) GO TO 440
      J=J+1
      ROOTRR(J)=ROOTR(I)
      ROOTRI(J)=ROOTI(I)
440   CONTINUE
```

```
      IDIMY=IDIMZ
500   CONTINUE
510   IKOUN1=N-KOUNT
      IF(IKOUNT.LT.1) GO TO 540
      IDIMY4=3
      Y4(1)=V2(1)
      Y4(2)=-V1(1)
      Y4(3)=1.0
      IF(IKOUNT.LT.2) GO TO 590
      X4(3)=1.0
      IDIMX4=3
      DO 530 I=2,IKOUNT
      X4(1)=V2(I)
      X4(2)=-V1(2)
      CALL PMPY(ZZ1,IDIMZZ,X4,IDIMX4,Y4,IDIMY4)
      DO 520 II=1,IDIMZZ
520   Y4(II)=ZZ1(II)
      IDIMY4=IDIMZZ
530   CONTINUE
540   IF(KOUNT.LT.1) GO TO 550
      IF(IKOUNT.LT.1) GO TO 570
      GO TO 590
550   DO 560 I=1,6
560   FD(I)=Y4(I)
      GO TO 800
570   DO 580 I=1,6
580   FD(I)=Y3(I)
      GO TO 800
590   CALL PMPY(FD,IDIMFD,Y4,IDIMY4,Y3,IDIMY)
800   CONTINUE
805   IF(K.NE.1000) GO TO 810
      MINA=N3
      GO TO 815
810   MINA=1
```

```
815   DO 10000 III=1,ITMA
      AX=RAN(DUM)
      IF(AX.GE.0.5) GO TO 820
      AX1=-1.0
      GO TO 830
820   AX1=1.0
830   IF(K.LT.1000) GO TO 850
      IF(K.EQ.1000) GO TO 850
      DO 840 I=2,N3
840   E(I-1)=E(I)
      E(N3)=AX1
      GO TO 860
850   E(III)=AX1
      KK=2**(III-1)
      IF(III.LE.M) MC=III
      MINA=III
860   Z=0.0
      DO 870 I=1,MC
      J=MINA+1-I
870   Z=Z+E(J)*FD(I)
      R1=RAN(DUM)
      R2=RAN(DUM)
      Z=Z+SIGMA*SQRT(-2.*ALOG(R1))*COS(PI*R2)
      DO 910 KL=1,L1
      T2=SS(2*KL-1)-Z
      T3=SS(2*KL)-Z
      IF(T2.GE.T3) GO TO 880
      AS(KL)=T3
      IPATH(KL)=PATH(2*KL)
      GO TO 890
880   AS(KL)=T2
      IPATH(KL)=PATH(2*KL-1)
890   T4=SS(2*KL-1)+Z
```

```
      T5=SS(2*KL)+2
      L2=L1+KL
      IF(T4.GE.T5) GO TO 900
      AS(L2)=T5
      IPATH(L2)=PATH(2*KL)+EE
      GO TO 910
900   AS(L2)=T4
      IPATH(L2)=PATH(2*KL-1)+EE
910   CONTINUE
      DO 920 I=1,L
      PATH(I)=IPATH(I)
920   SS(I)=AS(I)
      IF(III.LT.N3) GO TO 1000
      INDEX=1
      T6=SS(1)
      DO 930 I=2,L
      IF(T6.GE.SS(I)) GO TO 930
      INDEX=I
      T6=SS(I)
930   CONTINUE
      YY1=PATH(INDEX)-PATH(INDEX)/2*2
      IF(YY1.EQ.0) YY1=-1
      YY=YY1
      IF(E(1).NE.YY) IERR=IERR+1
      DO 940 I=1,L
940   PATH(I)=PATH(I)/2
      A(N1)=YY
      GO TO 960
950   A(N1)=AX1
960   DO 970 I1=1,2
      DO 970 I=1,N
      R1=RAN(DUM)
      R2=RAN(DUM)
      B(I,I1)=SIGMA*SQRT(-2.*ALOG(R1))*COS(PI*R2)
      DO 970 I2=1,N1
      J=N1+1-I2
970   B(I,I1)=B(I,I1)+A(J)*S(I,I2,I1)
```

```
9000  CONTINUE




      STOP
      END
      SUBROUTINE PCLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
      DIMENSION XCOF(20),COF(20),ROOTR(20),ROOTI(20)
      IFIT=0
      N=M
      IER=0
      IF(XCOF(N+1))10,25,10
10    IF(N) 15,15,32
15    IER=1
20    RETURN
25    IER=4
      GO TO 20
30    IER=2
      GO TO 20
32    IF(N-36) 35,35,30
35    NX=N
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 40 L=1,KJ1
      MT=KJ1+1-L
40    COF(MT)=XCOF(L)
C SET INITIAL VALUES
45    XO=0.00500101
      YO=0.01000101
      IN=0
50    X=XO
```

```
      XO=-10.0*YO
      YO=-10.0*X
  SET X AND Y TO CURRENT VALUES
      X=XO
      Y=YO
      IN=IN+1
      GO TO 59
55    IFIT=1
      XPR=X
      YPR=Y
  EVALUATE POLYNOMIAL AND DERIVATIVE
59    ICT=0
60    UX=0.0
      UY=0
      V=0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF(U) 65,130,65
65    DO 70 I=1,N
      L=N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      FI=I
      UX=UX+FI*XT*TEMP
      UY=UY-FI*YT*TEMP
      XT=XT2
70    YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF(SUMSQ) 75,110,75
75    DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
```

```
      Y=Y+DY
 78   IF(ABS(DY)+ABS(DX)-1.ØE-Ø5) 1ØØ,8Ø,8Ø
  STEP ITERATION COUNTER
 8Ø   ICT=ICT+1
      IF(ICT-5ØØ) 6Ø,85,85
 85   IF(IFIT) 1ØØ,9Ø,1ØØ
 9Ø   IF(IN-5) 5Ø,95,95
 95   IER=3
      GO TO 2Ø
1ØØ   DO 1Ø5 L=1,NXX
      MT=KJ1+1-L
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
1Ø5   COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF(IFIT) 12Ø,55,12Ø
11Ø   IF(IFIT)115,5Ø,115
115   X=XPR
      Y=YPR
12Ø   IFIT=Ø
122   IF(ABS(Y)-1.ØE-4*ABS(X)) 135,125,125
125   ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 14Ø
13Ø   X=Ø.Ø
      NX=NX-1
      NXX=NXX-1
135   Y=Ø.Ø
      SUMSQ=Ø.Ø
      ALPHA=X
      N=N-1
14Ø   COF(2)=COF(2)+ALPHA*COF(1)
```

```
145   DO 150 L=2,N
150   COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
155   ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF(SUMSQ) 160,165,160
160   Y=-Y
      SUMSQ=0.0
      GO TO 155
165   IF(N) 20,20,45
1000  CONTINUE
      END
      SUBROUTINE PMPY(Z,IDIMZ,X,IDIMX,Y,IDIMY)
      DIMENSION Z(20),X(20),Y(20)
      IF(IDIMX*IDIMY) 10,10,20
10    IDIMZ=0
      GO TO 50
20    IDIMZ=IDIMX+IDIMY-1
      DO 30 I=1,IDIMZ
30    Z(I)=0.
      DO 40 I=1,IDIMX
      DO 40 J=1,IDIMY
      K=I+J-1
40    Z(K)=X(I)*Y(J)+Z(K)
50    RETURN
      END
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